NUMERICAL STUDY OF SUPERSONIC FLOW PAST A CYLINDRICAL AFTERBODY

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Abstract

The near wake of a cylindrical afterbody aligned with a uniform Mach 2.46 flow has been numerically investigated using Reynolds-Averaged Navier-Stokes equations (k-epsilon two equation model) and Large Eddy Simulation (dynamic sub-grid scale eddy viscosity model). Mean flow field properties obtained from numerical simulations, such as axial velocity, pressure on the base surface has been compared with the experimental results. It has been found that k-epsilon model fails to predict the flow properties in the recirculation region where better agreement has been observed between the data obtained from LES and measurements. Data obtained from LES has been further analyzed to investigate the turbulent flow field in the wake region. Parameters like turbulent kinetic energy and primary Reynolds stress have been calculated and compared with the results obtained from an experiment in order to achieve a better understanding of the role of turbulence in the flow field.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$C_p$</td>
<td>Pressure co-efficient</td>
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<tr>
<td>$C_s$</td>
<td>Smagorinsky constant</td>
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<tr>
<td>$D$</td>
<td>Diffusivity</td>
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<tr>
<td>$E$</td>
<td>Total energy</td>
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<tr>
<td>$P_{inf}$</td>
<td>Free-stream pressure</td>
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<tr>
<td>$P_k$</td>
<td>Turbulent production</td>
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<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
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<tr>
<td>$R_o$</td>
<td>The radius of the cylindrical after-body</td>
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<td>$S_{ij}$</td>
<td>Mean flow strain rate tensor</td>
</tr>
<tr>
<td>$Sc$</td>
<td>Schmidt number</td>
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<tr>
<td>$T$</td>
<td>Temperature</td>
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<tr>
<td>$U$</td>
<td>Mean velocity</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>Species mass fraction</td>
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<tr>
<td>$c_p$</td>
<td>Heat capacity of a system under constant pressure</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\tau_{ij}$</td>
<td>Stress tensor</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Filter size</td>
</tr>
<tr>
<td>$\Delta \tau$</td>
<td>Pseudo-time step</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Source term in species transport equation</td>
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Superscripts

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<tr>
<th>Superscript</th>
<th>Meaning</th>
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<tr>
<td>^</td>
<td>Filtered variable</td>
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<tr>
<td>~</td>
<td>Favre averaged variable</td>
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<tr>
<td>–</td>
<td>Averaged variable</td>
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Subscripts

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<tr>
<th>Subscript</th>
<th>Meaning</th>
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<tr>
<td>1</td>
<td>Free stream condition</td>
</tr>
<tr>
<td>axial</td>
<td>Axial component of a vector</td>
</tr>
<tr>
<td>average</td>
<td>Time averaged variable</td>
</tr>
<tr>
<td>inf</td>
<td>Free stream condition</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean square</td>
</tr>
<tr>
<td>sgs</td>
<td>Sub grid scale component</td>
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Introduction

Objects like bullets, projectiles, missiles and rockets traveling at supersonic velocity are subjected to massive pressure drag due to the low-pressure region behind the base of these objects. Over the past years several active and passive techniques, such as boat-tailing, base cavity, base bleed and base burning have been developed to increase the base pressure and reduce overall drag on the bodies traveling at supersonic speed. Yet, in order to design optimal techniques for base pressure recovery, it is imperative to achieve a thorough understanding of the complex fluid dynamic processes that occur in the wake region.

In an attempt to understand the intricate physics of supersonic base flows, many experimental and numerical studies have been performed over the past few decades. Among these, most significant experimental studies have been performed by Herrin and Dutton [1]. They have performed an extensive study of mean and turbulent flow properties of the near wake region behind a cylindrical afterbody of 63.5 mm diameter, in perfect axial alignment with a Mach 2.44 flow. Their experimental facility was specifically designed to maintain the axial alignment while reducing the effects of support strings on the flow field. In our study, we have used the published data from their paper to validate our results from numerical simulations.

Several numerical studies have also been performed mostly focused on replicating the experimental data obtained by Herrin and Dutton [1]. Sahu [2] performed RANS study of base flow on 2D axisymmetric geometry and found that the two equation k-epsilon turbulence model) and LES have been assessed by comparing the results obtained from simulations with experimental data. Results obtained from the LES have been further analyzed to study the turbulent properties of the flow field.

Mathematical Modeling

Governing Equations

Favre-filtered governing equations for LES calculation have been enumerated below:

Continuity Equation:

\[ \frac{\partial}{\partial t} (\bar{\rho}) + \frac{\partial}{\partial x_j} (\bar{\rho} \cdot \hat{u}_j) = 0 \]  

(1)

Momentum Equation:

\[ \frac{\partial}{\partial t} (\bar{\rho} \cdot \hat{u}_j) + \frac{\partial}{\partial x_j} (\bar{\rho} \cdot \hat{u}_i \cdot \hat{u}_j) = -\frac{\partial}{\partial x_j} (\bar{\rho} \cdot \hat{u}_i) - \frac{\partial}{\partial x_j} (\tau_{iy}^{sgs}) \]  

(2)

Where,

\[ \tau_{ij} = \mu \left( \frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial \hat{u}_k}{\partial x_k} \delta_{ij} \]  

(3)

\[ (\tau_{ij})^{sgs} = \mu (\hat{u}_i \hat{u}_j) - \bar{\rho} \cdot \hat{u}_i \cdot \hat{u}_j = -\mu \left( \frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right) \]  

(4)

Energy Equation:

\[ \frac{\partial}{\partial t} (\bar{\rho} \cdot \hat{E}) + \frac{\partial}{\partial x_j} (\bar{\rho} \cdot \hat{E} \cdot \hat{u}_j) = \frac{\partial}{\partial x_j} (\hat{u}_j (-\hat{\rho} + \tau_{ij}^{sgs})) \]  

\[ + \frac{\partial}{\partial x_j} \left( \frac{k + \frac{\mu}{Pr} C_s}{Pr} \frac{\partial \hat{T}}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \sum_{s=1}^{N} h_s \frac{\hat{F}_s}{\hat{S}_s} \right) \]  

(5)

Where,
E = \frac{1}{2}u_i^2 \quad (6)

Species Transport Equation:
\frac{\partial}{\partial t} (\rho \cdot \dot{\xi} ) + \frac{\partial}{\partial x_j} (\rho \cdot \dot{u}_j ) = - \frac{\partial}{\partial x_i} \left[ (D + \frac{\mu}{S_{ij}}) \frac{\partial \dot{y}}{\partial x_i} \right] + \omega, \quad (7)

In this calculation air has been considered as a mixture of Nitrogen and Oxygen, thus having two species components.

Since its a non-reacting flow, ∇ · \omega = 0.

The contributions from the sub grid scales are calculated using following equations:
\mu \left[ \frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right] = 2 \cdot \rho \cdot \hat{u}_i \cdot \hat{S}_{ij} \quad (8)

Where,
\hat{S}_{ij} = \frac{1}{2} \left( \frac{\partial \hat{u}_i}{\partial x_j} - \frac{\partial \hat{u}_j}{\partial x_i} \right) \quad (9)

\nu_t = C_s^2 (\Delta)^2 \cdot \hat{S}_{ij} \cdot \hat{S}_{ij} \quad (10)

| \hat{S}_{ij} | = \frac{1}{2} \cdot \hat{S}_{ij} \cdot \hat{S}_{ij} \quad (11)

The coefficient Cs is calculated dynamically, as proposed by Geamano et al. [8] and locally averaged.

Numerical Scheme

A density based, fully coupled FVM based solver has been used to solve the governing equations. A second order Low Diffusion Flux Splitting Scheme has been used to discretize the convective terms (Edwards, [9]). All other spacial terms (i.e. diffusion terms) in the governing equations are discretized using second order central difference scheme, while the second order implicit temporal discretization is used. Moreover, the Low Mach number preconditioning (Weiss and Smith, [10]) is used in order to effectively capture the different flow regime in the domain. The parallel processing is done using Message Passing Interface (MPI) technique.

Computation Domain and Boundary Conditions

Dimensions of the computation domain and boundary conditions have been chosen in the super sonic base flow experiments performed by Herrin and Dutton [1]. The radius of the base is 31.75 mm and Mach number of the flow is 2.46 which leads to Reynolds number of 2.858 \times 10^6. Fig.1 illustrates dimension of the computation domain. The mesh has 1.97 million grid points. Since the region of interest is the near wake of the cylindrical afterbody, the fine mesh has been used in the near the base and walls. Further downstream and towards radially outside the domain the mesh has been stretched to optimize computation cost.

Boundary conditions applied to the domain are as follows:

| Pressure at Inlet | : 33094.28 Pa | Temperature at Inlet | : 134.202 K | Velocity at Inlet | : 566.66 m/s |

Adiabatic wall with no slip boundary condition has been applied on the cylindrical afterbody. Supersonic free-stream boundary condition with pressure, velocity and temperature same as the inlet has been applied on the outer boundary of the cylindrical computation domain, while at the outlet a non-reflecting convective outflow condition is used (Akselvoll and Moin, [11]). For LES simulation, the physical time step or Δt is kept 1 \times 10^{-6} corresponds to CFL number in order of 0.5. Simulations are carried out for several flow-through times, while the time averaging of the flow field is achieved over ~7 flow-through times to obtain better statistics.

Results and Discussion

Salient Features of the Flow Field

Despite the apparent simple geometry, supersonic flow past a cylindrical afterbody possess critical flow features like presence of expansion waves, shocks, strong shear layer, the interaction of unsteady vortices with reattachment shock. Fig.2 illustrates the salient features of the flow field in near wake of the cylindrical afterbody axially aligned with a supersonic flow of Mach Number 2.44. In Fig.2, it is discernable that the flow turns as Prandtl-Mayer expansion waves are formed at the base corner followed by a strong shear layer and further downstream the flow realigns itself with the axis after passing through a recompression region. A large recirculation bubble is formed immediately downstream of the base. The turbulent
boundary layer detaches from the body at the base corner and forms the shear layer. The free shear layer separates low velocity (subsonic) fluid in the recirculation region from the outer supersonic flow. Low-velocity fluid in the recirculation region is entrained and accelerated by the shear layer. Further, downstream the fluid in recirculation region encounters strong adverse pressure gradient and thus, is sent back from the stagnation point towards the base. The free shear layer is characterized by intensive turbulent mixing. The dynamics of energy transfer from the supersonic flow outside the recirculation region to the low-velocity flow inside the recirculation region is governed by the shear layer and thus flow properties in the wake region are greatly influenced by the shear layer.

**Mean Pressure and Velocity Profiles**

Time-averaged results obtained from LES, RANS (k-epsilon model) and results obtained from experiments (Herrin and Dutton, [1]) have been compared and thoroughly discussed in this section.

From an application point of view, the pressure at the base surface is the most crucial parameter in this study since the objective of the study is to understand how the flow physics is governing the pressure at the base. A dimensionless pressure coefficient $C_p$ has been calculated from pressure at the base of the cylindrical afterbody. The relation between base pressure and the pressure coefficient is given by Eq.12.

$$C_p = \frac{2 \left( \frac{P_{base}}{P_1} - 1 \right)}{\gamma M_1^2}$$  \hspace{1cm} (12)

In Fig.3 pressure coefficient obtained from LES, RANS, and experimental studies have been compared. It can easily be seen that k-epsilon model failed to predict the pressure distribution on the base surface where LES has predicted the base pressure with reasonable accuracy.

In Fig.4, time-averaged axial velocity distribution in streamwise direction obtained from LES, RANS and experiment have been compared. In Figs.5-9, time-averaged axial velocity distribution in the radial direction at five different locations downstream of the base have been plotted.

From Fig.4, it can be easily inferred that k-epsilon turbulence model fails to predict the location of the recirculation region but LES has been able to render a much better prediction of the location of the recirculation region.

Though LES has slightly over predicted the magnitude of maximum reverse flow in recirculation region and the location of the maximum reverse flow has also been predicted at a location further downstream, the length of the recirculation region predicted by LES is in good agreement with the experimental result. From the plots exhibiting time-averaged axial velocity distribution in the radial direction at different locations it can be seen that the prediction obtained from LES seems to be in excellent agreement with experimental results at the locations closer to the base. The width of the free shear layer has also been predicted with reasonable accuracy. But as we proceed further downstream, some discrepancies are encountered in the results obtained from LES.

From the plots, it can be clearly seen that k-epsilon model fails to predict the mean axial velocity at every location. In Fig.10 and Fig.11 the streamlines on a plane for LES and RANS (k-epsilon) cases have been plotted. By comparing these two figures it can be clearly seen that the distance of reattachment point from the base of cylinder predicted by RANS (k-epsilon) is much shorter than the distance that is being predicted by LES.

Similarly by comparing Fig.12 and Fig.13, it can be observed that K-epsilon model under predicts pressure in the near wake region. It is self-evident from these results that K-epsilon model failed to predict the highly turbulent flow field behind the cylindrical afterbody.

**Study of Turbulence in the Near Wake Flow Field**

Figures 14-15 exhibit the distribution of time-averaged turbulent kinetic energy and Figs.16-17 show the distribution of time-averaged principal Reynolds Shear Stress along radial direction at two different axial locations downstream of the base; one very close to the base and another one further downstream. From these plots, it can be concluded that prediction of turbulence in the shear layer obtained from LES studies fairly matches with the experimental results. In order to further analyze the turbulent properties of the flow field, contours of parameters like axial turbulence intensity, Turbulent Kinetic Energy (TKE), primary Reynolds shear stress and turbulence production ($P_k$) have been plotted in non-dimensional forms.
These parameters have been calculated using following equations.

\[
\text{Axial turbulence intensity} = \left( \frac{u'}{U_1} \right)_{\text{rms}} \quad (13)
\]

\[
TKE = k = \frac{1}{2} \times \left( \frac{u'^2 + \nu'^2 + w'^2}{\text{rms}} \right) \quad (14)
\]

\[
\text{Primary Reynolds Shear Stress} = \overline{uu'}/U_1^2 \quad (15)
\]

\[
P_k = -u'_i u'_j S_{ij} \quad (16)
\]

From the Figs.18-21 it can be inferred that thickness of the shear layer gradually expands in the downstream and intensity of turbulence, turbulent kinetic energy, magnitude of primary Reynolds Shear Stress and turbulence production also increases along the shear layer in the downstream direction and attains the maximum value in a region where the shear layer attaches.

The free shear layer seems to play a major role in the complicated dynamics of fluid the in the wake region of the cylindrical afterbody. Due to high shear stresses in the free shear layer, the turbulent production term attains a high value in the shear layer thus facilitating intensive turbulent mixing and energy transfer from the supersonic flow outside the recirculation region to the subsonic flow inside the recirculation region. Fluid in recirculation region, closer to the shear layer, has higher kinetic energy due to the turbulent mixing and penetrates through the adverse pressure gradient in the reattachment region. But fluid in the recirculation region, away from the shear layer, has less kinetic energy and fails to penetrate through the reattachment zone and is sent back towards the base after reaching the stagnation point. As a result, a low velocity and low-pressure recirculation zone are formed near the base.

**Study of Unsteady Flow Structures**

In Fig.23 the distribution of instantaneous vorticity magnitude at a different time, on \( z = 0 \) plane have been plotted. Highly unsteady three dimensional vortical structures are observed in the wake region of the cylindrical after-body. In plots of iso-vorticity surface (Fig.22) and instantaneous vorticity magnitude (Fig.23), it can be easily observed that the vortices are rapidly breaking up after passing through the reattachment region. Larger vortices are present in the downstream region where, in the recirculation region smaller vortices can be seen. From the sequence of contour plots presented in Fig.23, it can be seen how vortices are getting detached from the shear layer and forming mushroom like structures near the recompression region and eventually breaking up as they proceed further downstream. In the recompression region, due to presence of the recompression shock system and high temperature gradient, the recompression process is not an isentropic one thus the baroclinic contribution term \( \frac{\Delta \rho \times \nabla p}{\rho^2} \) in vorticity transport equation is nonzero. This might be the reason of formation of mushroom like structures near the recompression region.

**Conclusions**

A high Reynolds number supersonic base flow was numerically investigated using RANS and LES. It is found that the LES is able to predict the features of mean flow field and the turbulent properties of the flow field successfully and the predictions are found to be satisfactory. Unsteady flow structures such as large eddies in the downstream region, smaller vortices inside the recirculation region, mushroom-like structures near the reattachment region are also properly captured by the LES. On the other hand, the k-epsilon model fails to predict the flow field in the near wake region. The presence of strong shear layer and vortices interacting with the recompression shock system induces strong unsteadiness in the flow. This might be the reason why k-epsilon method fails to predict the turbulent mixing properly across the shear layer and eventually renders the wrong prediction of the recirculation region.

Incoming turbulent boundary layer detaches at the base corner and forms the free shear layer, thus, the incoming turbulent boundary layer will have some tangible effects on the shear stresses in the free shear layer and thus on the turbulence production. Alterations in the turbulence production in the free shear layer will have some effects on the fluid dynamics of the recirculation region. Thus there are scopes for further study by increasing the approach length of the geometry and resolving the approaching turbulent boundary layer further.

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sources available at IITK. This support is gratefully acknowledged.

**References**


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