THERMAL POSTBUCKLING OF HEATED COLUMNS OF VARIABLE CROSS-SECTION - A SIMPLE INTUITIVE FORMULATION

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Abstract

The thermal postbuckling of heated slender columns of variable cross-section (tapered columns) is evaluated by a simple intuitive formulation, when compared to the popular Rayleigh-Ritz method. The intuitive formulation requires only two parameters, namely, the mechanical equivalent of the thermal buckling load parameter and the constant tensile load parameter induced in the column, due to moderately large deflections, when the ends of the column are fixed axially. The thermal buckling load parameter of tapered columns is obtained from the Rayleigh-Ritz method. The symmetric boundary conditions, like hinged-hinged and clamped-clamped conditions, are considered for the tapered columns, which are consistent with the symmetry of the tapered column. The ratio of thermal postbuckling to buckling loads obtained from the simple intuitive formulation, varying with the taper and central deflection ratios, are presented for hinged-hinged and clamped-clamped tapered columns, which match very well with those obtained by Rayleigh-Ritz method, which is a validation of the simple intuitive formulation.

Keywords: Thermal Buckling; Thermal Postbuckling; Heated Columns; Variable Cross-section; Intuitive Formulation

Introduction

The structural engineers have now accepted, the importance of mechanical/thermal postbuckling loads, of structural members like columns, plates and shells, to achieve efficient and cost effective designs, in the fields of aerospace and automobile engineering. The emphasis is given to thermal postbuckling load, as this load is of an order of magnitude higher, when compared to mechanical postbuckling loads [1, 2]. The postbuckling problem is a geometrically nonlinear problem, where the deflections are moderately large, which is formulated by using the von-Karman nonlinear axial strain-displacement relations [1]. The solution of this nonlinear problem is relatively complex, when compared to the linear buckling problem that is formulated, by using the linear small deflection theory [3]. Hence, many researchers worked on the ther-
nal buckling and postbuckling of heated columns and other structural members, containing several complicating effects.

The main aim of the present investigation is to obtain the thermal postbuckling behavior of slender heated columns having a variable cross-section (tapered columns), by using a simple intuitive formulation. The tapered columns considered in this study, is a column with sinusoidally varying cross-section and is symmetric about the mid-point of the length of the column. For the purpose of a better comprehension of the intuitive formulation, to evaluate the thermal post-buckling behavior of slender tapered hinged-hinged (h-h) and clamped-clamped (c-c) columns, is clearly explained in this note. Some earlier studies on the buckling and post-buckling behavior of nonuniform columns can be seen in the papers by [4 - 6].

Analytical expressions for the thermal postbuckling load of heated tapered columns, for various values of taper ratios (TR) and ratios of the central deflection to the radius of gyration of the columns, at the end cross-sections, $x = 0$ and $x = L$, where $x$ is the axial coordinate and $L$ is the length of the tapered column. The development of the simple intuitive formulation which started around the last decade [7 - 10], to predict the thermal postbuckling behavior of heated tapered columns. The numerical results obtained from the simple intuitive formulation match very well with those obtained by the commonly used Rayleigh-Ritz (RR) method. This indicates the applicability of the simple intuitive formulation for tapered columns, developed earlier. The details of the intuitive formulation is presented in the following sections after the description of the configuration of the tapered column.

**Column with Sinusoidally Varying Cross-Section**

The configuration of the column of circular cross-section, with sinusoidally varying diameter, is shown in Fig. 1. The cross-sectional variation is symmetric about the mid-point of the of the length of the column $L \left( x = \frac{L}{2} \right)$. The diameters of the column at the mid-point is $(d_1 + d_2)$ and the ends are $d_1$ of the tapered column. The column is heated by a uniform temperature $\Delta T$ from its stress free temperature $T_{sf}$. The rise in temperature develops a mechanical equivalent of the thermal compressive load $P$, when at the two ends of the column, $x = 0$ and $x = L$, the axial displacement $u$ is fixed. This compressive load makes the heated tapered column to buckle and subsequently to have the postbuckling load. The boundary conditions are h-h and c-c, which are symmetric, so as to keep the structural system symmetric, for the purpose of thermal buckling and postbuckling analyses. From this configuration of the tapered column the diameter $d(x)$ and area of cross-section $A(x)$ and the area moment of inertia of the cross-section of the column $I(x)$ are given, by

$$d(x) = d_1 \left[ 1 + TR \sin\left( \frac{\pi x}{L} \right) \right]$$  \hspace{1cm} (1)

where, the TR is defined as $\frac{d_2}{d_1}$.

The expressions for $A(x)$ and $I(x)$ are given from that of $d(x)$

$$A(x) = A_1 \left[ 1 + TR \sin\left( \frac{\pi x}{L} \right) \right]^2$$  \hspace{1cm} (2)

and

$$I(x) = I_1 \left[ 1 + TR \sin\left( \frac{\pi x}{L} \right) \right]^4$$  \hspace{1cm} (3)

In Eqs.(1) and (3), the values $d_1$, $A_1$ and $I_1$ are the values at the ends of the tapered column, where the deflection/slope boundary conditions are specified.

**Evaluation of Ratio of Thermal Postbuckling to Buckling Loads - Intuitive Formulation**

It is well known that the minimization of the total potential energy, popularly known as the RR method or its finite element analogue [11], which is a powerful, reliable and robust numerical method that gives accurate results for the thermal buckling and postbuckling loads of heated columns. The much simpler intuitive formulation presented in this note, to predict the thermal postbuckling behavior, follows an entirely different path, from the relatively complex RR method. The parameters required in the intuitive formulation are only the thermal buckling load parameter $\lambda_b = \frac{P_b L^2}{EI_1}$, where $P_b$ is the thermal buckling load, the evaluation of which is discussed earlier and the tensile load parameter $\lambda_t = \frac{TL^2}{EI_1}$, where $T$ is the constant axial tensile load, which will be developed, due to moderately large deflections of the tapered column, when its end axial displacements $u$ are fixed.
The expression for $T$ is obtained, by following a simple procedure. In the original configuration of the column, the end axial displacement $u = 0$ at $x = 0$ and $L$. This configuration is changed, for the purpose of evaluation of $T$, which is an important part of the intuitive formulation, by keeping the condition at $x = 0$, as it is, and the condition on $u$ at $x = L$ is relaxed. An axial tensile load $T$ is applied at the relaxed end and this produces an outward displacement $u_{ot}$, which is obtained as

$$u_{ot} = \frac{T}{E} \int_{0}^{L} \frac{d x}{A(x)}$$  \hspace{1cm} (4)

If the column undergoes moderately large amplitudes, the inward displacement $u_{it}$ at the relaxed end [12], is given by

$$u_{it} = \frac{1}{2} \int_{0}^{L} \left( \frac{d w}{d x} \right)^2 d x$$  \hspace{1cm} (5)

By equating the magnitudes of $u_{ot}$ and $u_{it}$ given in Eqs.(4) and (5), the magnitude of the constant tensile load $T$, induced in the column is obtained, as

$$T = \frac{E}{2} \left[ \int_{0}^{L} \frac{d x}{A(x)} \right] \left[ \frac{L}{2} \left( \frac{d w}{d x} \right)^2 d x \right]$$  \hspace{1cm} (6)

Another major advantage of this simple intuitive formulation is that the admissible functions for the axial displacement $u$, though difficult but not impossible, are not required, and it is sufficient to have the admissible functions $w$, for the $h-h$ and $c-c$ columns. In the intuitive formulation, the ratio of the thermal postbuckling load parameter $\lambda_{pb}$ of the heated tapered columns, where $\lambda_{pb} = \frac{P_{pb} L^2}{E I_1}$ and $P_{pb}$ is the mechanical equivalent of the postbuckling load, corresponding to the postbuckling temperature $(\Delta T)_{pb}$, for a given value of the central deflection ratio $\beta = \frac{b}{r_1}$, where $r_1$ is the radius of gyration of the cross-section at the ends of the column, are evaluated from the following logical argument arguments. If the column is heated to a temperature $T_{pb}$, a mechanical equivalent of the constant compressive load $P$, and in the non-dimensional form $\lambda_{pb} = \frac{P L^2}{E I_1}$ is developed. Further increase of temperature up to a certain temperature, which is called the buckling temperature $(\Delta T)_{pb}$, and also is called as the bifurcation temperature (point), the compressive load produced in the column, which is called the thermal buckling load $P_{b}$, or in the non-dimensional form is the thermal buckling load parameter $\lambda_{b}$. At the bifurcation point the tensile load $T$ or the tensile load parameter $\lambda_{t}$ induced in the heated tapered column, which is a measure of the additional thermal load carrying capacity beyond the thermal buckling load of the heated tapered column, is zero. Any further increase of temperature from the buckling (bifurcation) temperature $(\Delta T)_{pb}$, the column will be in the thermal postbuckling regime, and a constant tensile load parameter $\lambda_{t}$, which is proportional to $\beta^2$, is induced, where $\lambda_{t}$ is greater than the thermal buckling load parameter $\lambda_{b}$, and the following simple equation can be written, as

$$\lambda_{pb} = \lambda_{b} + \lambda_{t}$$  \hspace{1cm} (7)

or

$$\frac{\lambda_{pb}}{\lambda_{b}} = 1 + \frac{\lambda_{t}}{\lambda_{b}}$$  \hspace{1cm} (8)

From Eq.(8), if the ratio $\frac{\lambda_{t}}{\lambda_{b}}$ is known, the thermal postbuckling ratio $\frac{\lambda_{pb}}{\lambda_{b}}$ can be evaluated, for the specified values of $TR$ and $\beta$, as $\lambda_{t}$ depends on $TR$ and $\beta^2$ and $\lambda_{b}$ depends on $TR$.

Another interesting point in the intuitive formulation is that if the value of $\lambda_{b}$ is not readily available, this formulation facilitates to determine the additional thermal postbuckling load beyond the buckling load parameter $\Delta \lambda_{pb} (= \lambda_{pb} - \lambda_{b})$ from the equation given below, by subtracting Eq.(7) by $\lambda_{b}$, the following equation is obtained, as

$$\lambda_{pb} - \lambda_{b} = \Delta \lambda_{pb} = \lambda_{t}$$  \hspace{1cm} (9)
As mentioned earlier that $\lambda_t$ is a measure of the additional load that can be taken by the tapered column for a specified $\beta$ and TR. It is to be noted that $\lambda_t$ gives a quick assessment of the additional thermal load carrying capacity of the heated tapered column beyond its thermal buckling load.

**Brief Presentation of Thermal Buckling and Post-buckling of Tapered Column - RR Method**

The RR method to evaluate the thermal buckling and consequently postbuckling of tapered columns, is very briefly presented, in this section, the basic equations of the RR method are given here, which will help to have a better appreciation of the simple intuitive formulation. In the RR method the total potential energy of the tapered column $\pi$ is minimized with respect the unknown coefficients of the admissible functions used for the deflection $w(x)$ and axial displacement $u(x)$. The total potential energy $\pi$ of the tapered column is given, by

$$\pi = U_{NL} + U_L - W$$ \hspace{1cm} (10)

where, $U_{NL}$ is the nonlinear strain energy, $U_L$ is the linear strain energy and $W$ is the work done by the mechanical equivalent of the thermal load $P$ in the lateral direction.

Following [13], the value of $P$ is obtained, as

$$P = E L \alpha \Delta T \int_0^L \frac{dx}{A(x)}$$ \hspace{1cm} (11)

where, $E$ is the Young’s modulus $\alpha$ is the coefficient of the linear thermal expansion of the material of the tapered column. The expressions for $U_{NL}, U_L$ and $W$ are obtained [3], as

$$U_{NL} = \frac{E}{2} \int_0^L A(x) \varepsilon_x^2 \, dx$$ \hspace{1cm} (12)

$$U_L = \frac{E}{2} \int_0^L I(x) \psi_x^2 \, dx$$ \hspace{1cm} (13)

and

$$W = \frac{P}{2} \int_0^L \left( \frac{d w}{dx} \right)^2 \, dx$$ \hspace{1cm} (14)

In Eq.(12), $\varepsilon_x$ represents the von-karman nonlinear strain-displacement relation [1], given by

$$\varepsilon_x = \frac{du}{dx} + \frac{1}{2} \frac{d^2 w}{dx^2}$$ \hspace{1cm} (15)

and in Eq.(13), $\psi_x = -\frac{d^2 w}{dx^2}$ is the curvature, which is linear. The expression for $W$ is the standard one [3].

In the RR method, it is necessary that the admissible functions for the deflection $w$ and the axial displacement $u$, have to satisfy the essential boundary conditions. The most commonly used one term trigonometric admissible functions for $w$, applicable for one dimensional problems like column are taken from Leissa [14], as

$$w = b \sin \frac{\pi x}{L}$$ for the $h-h$ column, and

$$w = \frac{b}{2} \left( 1 - \cos \frac{2 \pi x}{L} \right)$$ for the $c-c$ column. Similarly, the admissible functions for $u = a \sin \frac{2 \pi x}{L}$ and $u = a \sin \frac{4 \pi x}{L}$, for the $h-h$ and $c-c$ columns, respectively [15]. The coefficients, in the context of buckling analysis can be treated as the undetermined coefficients. By minimizing the total potential energy $\pi$, after neglecting the nonlinear strain energy $U_{NL}$, as

$$\frac{d \pi}{db} = 0$$ \hspace{1cm} (16)

where the linear total potential energy does not contain $a$. As such, from Eq.(16), after evaluating the integrals and after simplification, the thermal buckling parameter

$$\lambda_t = \frac{P_b L^2}{E I_1}$$, where $P_b$ is the thermal buckling load, can be obtained.

In the same way, if the nonlinear strain energy is not neglected in $\pi$, the minimization process gives the two equations, as

$$\frac{d \pi}{db} = 0$$ \hspace{1cm} (17)

and

$$\frac{d \pi}{da} = 0$$ \hspace{1cm} (18)

From Eqs.(17) and (18), two nonlinear algebraic equations will be obtained, the solution of which gives the
thermal postbuckling parameter \( \lambda_{pb} = \frac{P_{pb} L^2}{E I} \) of the tapered columns. It is to be noted that the values of the \( b \) and \( a \) are the undetermined coefficients in the minimization process, and in the context of thermal postbuckling, the coefficient \( a \) gets eliminated and the coefficient \( b \) represents the deflection of the column at the mid-point along the length of the column. By knowing the expressions/numerical values \( \lambda_{pb} \) and \( \lambda_b \), the thermal postbuckling results can be obtained as \( \frac{\lambda_{pb}}{\lambda_b} \), which is the standard form.

It can be observed that deriving the expression for the thermal buckling load is simpler by orders of magnitudes when compared to the same for thermal postbuckling load, because of the existence of nonlinear axial energy term \( U_{NL} \) in the total potential energy \( \pi \).

**Numerical Results and Discussion**

As has been already mentioned, the simple intuitive formulation requires the thermal buckling and constant tensile load parameters \( \lambda_b \) and \( \lambda_t \), to evaluate the ratio of the thermal postbuckling to buckling load parameters \( \frac{\lambda_{pb}}{\lambda_b} \), for different values of TR and \( \beta \), of the h-h and c-c tapered columns with the sinusoidally varying symmetric cross-section. The value of \( \lambda_b \) is obtained, by applying the classical RR method. It is to be noted that all the integrals appearing in both the intuitive formulation and RR method are evaluated numerically.

The values of the thermal buckling load parameters \( \lambda_b \) are presented in Table-1, for both the h-h and c-c columns, for different values of TR. It is to be noted that the admissible functions for \( w \) are exact, for the uniform column for the boundary conditions considered, and these functions are approximate for tapered columns, where the approximation increases as the value of TR increases. Hence, in the present study the value of TR is restricted to 0.5 only. The values of \( \lambda_b \) are exact [3], for both the h-h and c-c columns, and are approximate within the tolerable limits of engineering accuracy, as the TR increases up to 0.5. As the TR increases, the stiffness of the tapered column increase, and hence, the values of \( \lambda_b \) increase with increasing TR. Another point to be noted is that the stiffness of h-h column is lower than that of c-c column, hence, the values of \( \lambda_b \) for the h-h column is lesser than c-c column for any specified value of TR, in the present study.

<table>
<thead>
<tr>
<th>TR</th>
<th>h-h Column</th>
<th>c-c Column</th>
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<tbody>
<tr>
<td>0.0</td>
<td>9.8696</td>
<td>39.4784</td>
</tr>
<tr>
<td>0.1</td>
<td>13.6922</td>
<td>50.1202</td>
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<tr>
<td>0.2</td>
<td>18.5725</td>
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<td>24.6937</td>
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<td>101.0588</td>
</tr>
<tr>
<td>0.5</td>
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<td>126.1768</td>
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</table>

The values of \( \frac{\lambda_{pb}}{\lambda_b} \) are presented in Table-2 for h-h tapered column and Table-3 for the c-c column, respectively, for different values of TR and \( \beta \). For the value of this ratio also, the limitation on TR mentioned for thermal buckling load parameter holds good. Further, there is one more limitation on the central deflection ratio \( \beta \). As the basic formulation used to predict the thermal postbuckling is due to von-Karman [1], is valid for moderately large deflections, the value of the specified \( \beta \) has to be moderately large. Hence in the present study, because of the limitations on the upper limits of TR and \( \beta \), which are fixed at 0.5 and 1.0. For the uniform column (TR = 0), the values of \( \beta \) for both the h-h and c-c columns are exact, as given

<table>
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<th>TR</th>
<th>\beta</th>
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<tr>
<td>0.0</td>
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<tr>
<td></td>
<td>(1.0156)*</td>
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<tr>
<td>0.1</td>
<td>1.0127</td>
</tr>
<tr>
<td></td>
<td>(1.0105)</td>
</tr>
<tr>
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<td></td>
<td>(1.0105)</td>
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<td>(1.0087)</td>
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<td>0.4</td>
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<td></td>
<td>(1.0073)</td>
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<tr>
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</tr>
</tbody>
</table>

* Numbers in the parentheses are from RR Method
Another interesting phenomenon, which is observed from the results of many researchers on thermal postbuckling is that the ratios of $\frac{\lambda_{pb}}{\lambda_b}$ decrease as the stiffness of the tapered columns, which increase either with $TR$ or with the boundary conditions. However, this ratio increases with the increase of $\beta$, as the tensile load parameter $\lambda_t$ is proportional to $\beta^2 = \left(\frac{b}{r_1}\right)^2$. Surprisingly, this opposite trend of the ratio $\frac{\lambda_{pb}}{\lambda_b}$, for a given $\beta$ can be seen by a close observation of the numerical results presented in Tables 1 and 2. Finally, the numerical results for the ratio of $\frac{\lambda_{pb}}{\lambda_b}$ obtained by the simple intuitive formulation match very well with those obtained by RR method, which is very briefly discussed in this note, validating the applicability of the simple intuitive formulation.

### Conclusions

The efficacy of the intuitive formulation to predict the thermal postbuckling behavior of columns with symmetric sinusoidally varying cross-section is shown, based on the numerical results presented. In the present study only the $h$-$h$ and $c$-$c$ symmetric boundary conditions are considered which are consistent with the symmetric taper, to make the structural system symmetric. The value of the thermal buckling load parameter is evaluated by using the RR method. The procedure for evaluating the constant tensile load parameter is presented clearly, in this note. The numerical results in terms of the buckling load parameter and the ratios of thermal postbuckling to buckling loads are given digitally. The following conclusions, based on the numerical results are given below:

- The thermal buckling load parameters, for the limiting case of uniform columns match exactly with the literature values, for both the $h$-$h$ and $c$-$c$ columns.
- As the stiffness of the columns of sinusoidally varying cross-section increases with the taper ratio, the thermal buckling load parameter increases as the taper ratio increases.
- For a specified taper ratio, the stiffness of the $h$-$h$ column is lesser than that of the $c$-$c$ column, as such the thermal buckling load parameter for the $h$-$h$ column is lower than that of the $c$-$c$ column.
- The ratio of the thermal postbuckling load to buckling load, from now onwards called as the simply ratio, which is dependent on the taper ratio and square of the central deflection ratio.
- This ratio for the uniform columns matches exactly with the literature values for a specified value of the central deflection ratio.
- This ratio for the tapered column decreases as the taper and central deflection ratios increase ratio increase.
- For the $h$-$h$ tapered column, this ratio increases and for the $c$-$c$ column this ratio decreases, for a specified taper and central deflection ratios.
- The aforementioned two items means that this ratio behave in an opposite way when compared to the thermal buckling load parameter.
- These ratios match very well with those obtained by applying Rayleigh-Ritz method, with negligible difference, even for the extreme values of taper and central deflection ratios, namely, 0.5 and 1.0, for both the $h$-$h$ and $c$-$c$ tapered columns. This validates the simple intuitive formulation, for tapered columns presented in this note.

The intuitive formulation presented is general and hence, can be used for other similar structural members, without much of difficulty.

### Table-3 : Values of $\frac{\lambda_{pb}}{\lambda_b}$ for $c$-$c$ Columns - Intuitive Formulation

<table>
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* Numbers in the parentheses are from RR Method
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References


Fig.1 Configuration of Sinusoidally Varying Tapered Columns (BCs - Boundary Conditions on Deflection)