LARGE DEFLECTION ANALYSIS OF RHOMBIC SANDWICH PLATES WITH ORTHOTROPIC CORE

Gora Chand Chell*, Subrata Mondal** and Goutam Bairagi+

Abstract

This paper represents nonlinear analysis of rhombic sandwich plates with orthotropic core under uniform load. Banerjee’s hypothesis [1] involving a new form of energy expression in the total potential energy of the system has been employed. As a consequence the differential equation is decoupled keeping intact its nonlinear character. The aim of the present study is to analyze the nonlinear behaviour of rhombic sandwich plates with orthotropic core under uniform load for different skew angles. The results have been obtained both for movable and immovable edges from a single cubic equation. Numerical results (central deflection vs. load) have been computed and compared with known results for square sandwich plates only. Results for different skew angles are believed to be new.

Nomenclature

E = Young’s modulus
q, q0 = uniform load
v = Poisson’s ratio
a = size of the plate
t = face thickness
h = core thickness
Gxz = shear modulus in xz plane
Gyz = shear modulus in yz plane
u, v, w = displacement in x, y, z direction respectively
λ = constant depending on the Poisson’s ratio of the plate materials
I = first invariant of average face strain
θ = skew angle
r = u^2 - u^1
s = v^2 - v^1
r, s = components of the in plane displacement of u and v

Introduction

Nonlinear analysis of sandwich plate is interesting to design engineers for their wide application in the practical field. Important papers in this field are due to Chien and Chen [2], where the authors successfully carried out the analysis on the effect of initial stresses of nonlinear vibration of laminated plates on an elastic foundation. Nonlinear partial differential equation based on Mindlin plate theory are derived for nonlinear vibration of laminated plates.

It is well known that a good number of structural design utilizes sandwich type construction in the fabrication of major structural components. A high strength to weight ratio is achieved by combining a relatively thick light weight core with two thin high strength faces. The problem of large deflection of sandwich plates has been investigated by several authors, among which works of Reissner [3], Wang [4], Hoff [5] and Eringen [6] need special mention. Reissner [3] presented an exact analysis of finite deflection of sandwich plates. Wang [4] gave a general theory of large deflections of sandwich plates and shells. Hoff [5] and Eringen [6] each developed a theory of bending and buckling of sandwich plates. All these investigations are, however, confined to rectangular sandwich plates under mechanical loading only. Kamiya [7] employed Berger’s well known technique to solve nonlinear problems of sandwich plates using a new set of governing equation with a correction factor. This work has been restricted to a particular plate geometry.


Some more interesting papers on sandwich structure could be located [12-15] where nonlinear analyses have been carried out elegantly and these papers are attractive to design engineers.

Interesting papers on sandwich plates with orthotropic core are due to Yeh and Chen [16], Wu and Yu [17], Silva and Hunt [18] and Chakrabarti, Mukhopadhyay and Bera [19].

Yeh and Chen [16] carried out finite element dynamic analysis of orthotropic rectangular sandwich plates with an electro rheological fluid core layer. The finite element method and Hamilton’s principle are employed to derive the finite element equations of motion for the orthotropic sandwich plate. Wu and Yu [17] used a simple approach to obtain the natural frequencies for rectangular corrugated-core sandwich plates with all edges simply supported. Silva and Hunt [18] worked on interactive buckling analysis for sandwich structures with orthotropic core. Chakrabarti, Mukhopadhyay and Bera [19] presented nonlinear stability of a shallow unsymmetrical heated orthotropic sandwich shell of double curvature with orthotropic core.

Dutta and Banerjee [20] carried out the analysis of large deflections of sandwich plates with orthotropic core. Chell, Mondal and Bairagi [21] analyses large deflection of rhombic sandwich plates placed on elastic foundation. Literature on large deflection analysis of skew sandwich plates demands special attention because of their wide application in space industry. So far it is known that no paper could be located where nonlinear analysis of skew sandwich plates with orthotropic core has been investigated.

The aim of the present study is to use a set of uncoupled differential equations in oblique coordinates to analyze nonlinear behaviours of rhombic sandwich plates with orthotropic core under uniform loading using Banerjee’s Hypothesis [1]. To obtain the central deflection \( w_0 \) vs load \( q_0 \) Galerkin technique has been used. Numerical results thus obtained for different skew angles have been plotted in graph. Results have been compared with the results obtained by Dutta and Banerjee [20] for square sandwich plates only. The numerical results of the non-linear behaviours for different skew angles of rhombic sandwich plates with orthotropic core are believed to be new.

**Governing Equation**

We consider a rhombic sandwich plate as shown in Fig.1a with an orthotropic core as well as isotropic upper and lower faces; while the faces respond to the bending and membrane action of the plate, the core is assumed to transfer only shear deformation. Moreover the thickness of upper and lower faces is sufficiently thin in comparison with core thickness \( h \gg t \) to ignore a variation of shear in the thickness direction of the faces.

Under mechanical loading the governing equations for sandwich plates with orthotropic core in rectangular Cartesian coordinate are given by [1],

\[
\frac{E t}{2 (1 - v^2)} \left( \frac{\partial^2 r}{\partial x^2} + v \frac{\partial^2 s}{\partial x \partial y} \right) + \frac{E t}{4 (1 + v)} \left( \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 s}{\partial x \partial y} \right) + \left( \frac{\partial w}{\partial x} - \frac{r}{h} \right) G_{xz} = 0
\]

\[
\frac{E t}{2 (1 - v^2)} \left( \frac{\partial^2 s}{\partial x^2} + v \frac{\partial^2 r}{\partial x \partial y} \right) + \frac{E t}{4 (1 + v)} \left( \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 r}{\partial x \partial y} \right) + \left( \frac{\partial w}{\partial y} - \frac{s}{h} \right) G_{yz} = 0
\]

\[
\frac{2 E t u}{1 - v^2} \left( \frac{\partial^2 w}{\partial x^2} + v \frac{\partial^2 w}{\partial y^2} \right) + \frac{E t h}{1 - v} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 - \left( G_{xz} \frac{\partial r}{\partial x} + G_{yz} \frac{\partial s}{\partial y} \right) + q = 0
\]

\[
\frac{3 E t u}{1 - v^2} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)
\]

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \).
For a skew plate, let us adopt a system of oblique coordinates \((x_1, y_1, \theta)\) as shown in Fig.1b. Clearly \(x = x_1 \cos \theta \) and \(y = y_1 + x_1 \sin \theta \) are the coordinate transformation equations. Hence the partial differential operators become

\[
\frac{\partial}{\partial x} = \sec \theta \left( \frac{\partial}{\partial x_1} - \sin \theta \frac{\partial}{\partial y_1} \right) \\
\frac{\partial}{\partial y} = \frac{\partial}{\partial y_1}
\]

Putting the above transformations in Eqs. (1, 2, 3 and 4), we get the following set of differential equations:

\[
\frac{Et}{2(1-v^2)} \left[ \sec^2 \theta \left( \frac{\partial^2 r}{\partial x_1^2} - 2 \sin \theta \frac{\partial^2 r}{\partial x_1 \partial y_1} \right) + \tan^2 \theta \frac{\partial^2 r}{\partial y_1^2} + \frac{\partial^2 r}{\partial x_1 \partial y_1} \right] + \frac{Et}{4(1+v)} \left[ \frac{\partial^2 s}{\partial y_1^2} + \sec \theta \left( \frac{\partial^2 s}{\partial x_1 \partial y_1} - \sin \theta \frac{\partial^2 s}{\partial y_1^2} \right) \right] + \sec \theta \left( \frac{\partial w}{\partial x_1} - \sin \theta \frac{\partial w}{\partial y_1} \right) G_{xz} = 0 \\
\frac{Et}{2(1-v^2)} \left[ \sec^2 \theta \left( \frac{\partial^2 r}{\partial x_1^2} - 2 \sin \theta \frac{\partial^2 r}{\partial x_1 \partial y_1} \right) + \tan^2 \theta \frac{\partial^2 r}{\partial y_1^2} + \frac{\partial^2 r}{\partial x_1 \partial y_1} \right] + \sec \theta \left( \frac{\partial ^2 s}{\partial x_1 \partial y_1} - \sin \theta \frac{\partial ^2 s}{\partial y_1^2} \right) G_{yz} = 0
\]

\[
\frac{2Et \mu}{(1-v^2)} \left[ \sec^2 \theta \left( \frac{\partial^2 w}{\partial x_1^2} - 2 \sin \theta \frac{\partial^2 w}{\partial x_1 \partial y_1} \right) + \tan^2 \theta \frac{\partial^2 w}{\partial y_1^2} + \frac{\partial^2 w}{\partial x_1 \partial y_1} \right] + \frac{Et \lambda}{1-v^2} \left[ \frac{\partial w}{\partial x_1} \right]^2 + 2 \sec \theta \left( \frac{\partial w}{\partial x_1} - \sin \theta \frac{\partial w}{\partial y_1} \right)^2 + \frac{\partial w}{\partial y_1} + 4 \sec \theta \left( \frac{\partial w}{\partial x_1} - \sin \theta \frac{\partial w}{\partial y_1} \right) G_{xz} \frac{\partial w}{\partial x_1} \left( \frac{\partial w}{\partial y_1} \right)^2 + \frac{\partial^2 w}{\partial y_1^2} + \frac{h}{G_{xz}} \left[ \sec^2 \theta \left( \frac{\partial^2 w}{\partial x_1^2} - 2 \sin \theta \frac{\partial^2 w}{\partial x_1 \partial y_1} \right) + \tan^2 \theta \frac{\partial^2 w}{\partial y_1^2} \right] + G_{yz} \frac{\partial^2 w}{\partial y_1^2} \right]
\]

\[
\quad + h \left[ G_{xz} \sec \theta \left( \frac{\partial r}{\partial x_1} - \sin \theta \frac{\partial r}{\partial y_1} \right) + G_{yz} \frac{\partial w}{\partial y_1} \right] + q = 0
\]
$I^m = \frac{\partial}{\partial x} \left( u + u^r \right) + v \frac{\partial}{\partial y} \left( v + v^r \right)$

$+ \frac{1}{2} \left[ \sec \theta \left( \frac{\partial w}{\partial x_1} - \sin \theta \frac{\partial w}{\partial y_1} \right)^2 \right] + v \frac{1}{2} \left( \frac{\partial w}{\partial y_1} \right)^2$  \hspace{1cm} (8)

**Analysis**

Let us assume

$$w = w_0 \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$  \hspace{1cm} (9)

$$r = \bar{r} \cos \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$  \hspace{1cm} (10)

$$s = \bar{s} \sin \frac{\pi x_1}{a} \cos \frac{\pi y_1}{a}$$  \hspace{1cm} (11)

$$q = q_0 \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$  \hspace{1cm} (12)

this form of $w$ clearly satisfies the required simply supported edge conditions.

It is to be noted that for movable edge conditions $I^m = 0$.

Putting (9), (10), (11), (12) and (13) in Eq. (7) and remembering the values of $r$ and $s$ obtained from Eq. (5) and (6), we get the error function $\varepsilon (x, y)$, Galerkin’s Technique requires

$$\int \int \varepsilon (x, y) \, dx \, dy = 0$$  \hspace{1cm} (14)

Evaluating the cubic integrals in (14) we get the following form of cubic equation, determining the central deflection $w_0/h$:

$$A \left( \frac{w_0}{h} \right)^3 + B \left( \frac{w_0}{h} \right) + C = 0$$  \hspace{1cm} (15)

where,

$$A = \left[ \frac{\pi^4 t}{4 (1 - v^2)} \left( \sec^2 \theta + \tan^2 \theta + v \right)^2 + \frac{\pi^4 t}{h (1 - v^2)} \right]$$

$$B = \left[ \frac{\pi^2 a^2}{E h^2} \left( G_{xx} \left( \sec^2 \theta + \tan^2 \theta \right) + G_{yz} \right) \right.$$

$$+ \frac{\pi^2 a^2}{E h^3} \left( a \frac{K_1}{K_2} G_{xx} \sec \theta - \frac{K_1}{K_2} G_{yz} \right]$$

and $C = -\frac{q_0 a^4}{E h^4}$

$$K_1 = \frac{Et \pi^2 (3-v) G_{xz}}{4a^2 (1-v^2)} - \left[ \frac{Et \pi^2 (3-v) G_{xz}}{4a^2 (1-v^2)} + \frac{G_{xz}}{h} \right] G_{yz}$$
\[ K_2 = \frac{E^2 t^4}{16a^4 (1-v^2)} \left[ \frac{E t \pi^2 (3-v)}{4a^2 (1-v^2)} + \frac{G_x z_x}{h} \right] \left[ \frac{E t \pi^2 (3-v)}{4a^2 (1-v^2)} + \frac{G_x z_x}{h} \right] \]

\[ K_3 = \frac{E t \pi^2}{4a^2 (1-v^2)} + \frac{G_x z_x}{h} \left[ \frac{G_x z_x}{a} - \frac{E t \pi^2 G_x z_x}{4a^2 (1-v^2)} \right] \]

Numerical Results

Tables (1 and 2) and Figs. 2 and 3 show the results for the central deflection parameter for different load function of a rhombic sandwich plates with orthotropic core under uniformly distributed load with the following dimensions and material properties [23]:

- \( E = 69000 \times 10^6 \, \text{N/m}^2 \)
- \( t = 0.635 \times 10^{-3} \, \text{m} \)
- \( v = 0.3 \)
- \( G_x z_x = 455 \times 10^6 \, \text{N/m}^2 \)
- \( \lambda = v = 0.09 \)
- \( G_{yz} = 205 \times 10^6 \, \text{N/m}^2 \)
- \( h = 1.7135 \times 10^{-2} \, \text{m} \)

### Table-1: Nonlinear Analysis (Immovable Edge Conditions)

<table>
<thead>
<tr>
<th>( \frac{q_0 a^4}{E h^2} )</th>
<th>For ( \theta = 0^\circ ) a = 0.254 m</th>
<th>For ( \theta = 30^\circ ) a = 0.273 m</th>
<th>For ( \theta = 45^\circ ) a = 0.302 m</th>
<th>For ( \theta = 60^\circ ) a = 0.359 m</th>
</tr>
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<tr>
<td>( \frac{W_0}{h} ) (known) [20]</td>
<td>( \frac{W_0}{h} ) (calculated)</td>
<td>( \frac{W_0}{h} ) (calculated)</td>
<td>( \frac{W_0}{h} ) (calculated)</td>
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### Table-2: Nonlinear Analysis (Movable Edge Conditions)

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<th>( \frac{q_0 a^4}{E h^2} )</th>
<th>For ( \theta = 0^\circ ) a = 0.254 m</th>
<th>For ( \theta = 30^\circ ) a = 0.273 m</th>
<th>For ( \theta = 45^\circ ) a = 0.302 m</th>
<th>For ( \theta = 60^\circ ) a = 0.359 m</th>
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</thead>
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<tr>
<td>( \frac{W_0}{h} ) (known) [20]</td>
<td>( \frac{W_0}{h} ) (calculated)</td>
<td>( \frac{W_0}{h} ) (calculated)</td>
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Numerical results have been calculated using Eq. (15) by considering size of the plate ‘a’ as a variable and area of the plate as constant. For $\theta = 0^\circ$, Eq. (15) becomes same with Eq. (38) [20].

**Conclusion**

The proposed differential equations are uncoupled and thus simple. From the same Cubic Eq. (15) for $\left(\frac{w_0}{h}\right)$, the results of immovable and movable edge conditions can be obtained. Numerical results for rhombic sandwich plates with orthotropic core are obtained for different skew angles and presented those in Tables (1 and 2). For square sandwich plate the results are compared with those of known results [20] and are in good agreement. The results for the other different skew angles are believed to be completely new. The numerical results presented in tables for different skew angles, offer an interesting observation. As $\theta$ increases i.e., as the plate tends towards rhombic shape, the deflection $\left(\frac{w_0}{h}\right)$ decreases i.e., stress decreases.

This is quite expected because with the increase of skew angles, the plate offers more rigid structure. The effective span of the plate is reduced as $a \cos \theta$, so the centre of gravity of the plate is nearer to the edges for which it behaves more rigid structure and less deflected.

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**References**


