A SIMPLE METHOD TO PLOT PHOTOELASTIC FRINGES AND PHASEMAPS FROM FINITE ELEMENT RESULTS

B. Neethi Simon* and K. Ramesh*

Abstract

Plotting of photoelastic fringes and phasemaps from Finite Element (FE) results enables whole-field visual comparison with experimental results. There have been studies to plot isochromatic and isoclinic fringe contours and phasemaps from FE results earlier, using special, stand-alone, post-processing software, developed on different platforms. In this paper, to simulate photoelastic fringe contours and phasemaps realistically, a new strategy that uses the standard post-processor functionalities available in a commercial FE package is proposed and validated with experimental results. The simplified approach also comes in handy for visualizing the photoelastic fringe patterns in transient problems.

Keywords: Digital photoelasticity, Plotting of contours, FE validation, Isochromatics, Isoclinics, Inconsistent zones, Transient problems

Introduction

Photoelasticity is a whole field optical experimental technique which provides the information of principal stress difference ($N$) and principal stress direction ($θ$) in the form of fringe contours called isochromatics and isoclinics [1, 2]. In conventional photoelasticity, one records only fringe contours, and there have been studies to plot isochromatic and isoclinic fringe contours from FE results earlier to enable whole-field visual comparison with experimental results. Ramesh and his co-workers [3-5] were the first to present a practical method of plotting isochromatic fringe contours from two-dimensional (2D) FE results by using a scanning scheme. Ragulskis and Ragulskis [6] proposed a smoothing procedure to obtain individual isochromatic and isoclinic fringe contours acceptable for hybrid experimental and numerical analysis. Umezaki and Terauchi [7] attempted to extract only isotropic points of the isoclinic field in structures from a combination of photoelastic experiment and FE analysis. All of these studies used special software on various platforms, created for the purpose of post-processing the FE results as desired.

With the advent of digital photoelasticity, the whole-field estimation of photoelastic parameters has become faster and more accurate and phase-shifting techniques are commonly used for quantitative estimation of photoelastic parameters. Phasemaps are grayscale images which represent the variation of the photoelastic parameters over the domain. Recently, Ashokan and Ramesh [8] developed a special post-processing software on VC++ platform to plot isoclinic and isochromatic phasemaps from 2D FE results using an eigen function approach.

When the FE results are post-processed using stand-alone post-processing software, an underlying constraint is that only specific element types for which the software has been developed can be used for discretising the model domain for the FE analysis. Moreover the process requires the FE results to be input to the post-processor in a specific format. This becomes cumbersome and inefficient while handling transient problems, as for each time step, the input file needs to be created separately.

To overcome these difficulties, in this paper, a new strategy that uses the post-processor functionalities available in a commercial FE package (Abaqus v6.7) is proposed to plot photoelastic fringes and phasemaps; such facilities may not have been available earlier which has prompted the earlier researchers to develop their own post-processing software. To mimic various fringe contours/phasemaps, new colour spectrums for plotting are arrived at. The isochromatic and isoclinic phasemaps are simulated from the FE results.

The proposed methodology is validated for the benchmark problem of a ring under diametral compression. It is

*Department of Applied Mechanics, Indian Institute of Technology Madras, Chennai-600 036, India, Email: kramesh@iitm.ac.in
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then applied to a more complex problem to demonstrate the use of the methodology with different element types. A transient thermal stress problem [9] is used to visualize transient photoelastic fringes and the effectiveness of the proposed methodology in handling transient problems is thus demonstrated. For the sake of completeness, brief introductions to the experimental photoelastic fringe contours and phasemaps are also given.

**Photoelastic Fringes**

Photoelastic fringes (isochromatics) are contours of constant principal stress difference. When a loaded photoelastic model is seen in a circular polariscope, using a monochrome light source, there are alternating bright and dark fringes; when white light source is used, the fringes are multicoloured, with colours appearing in a specific order. One is interested in simulating these fringe patterns realistically from FE results.

**Experimental Photoelastic Fringes**

To illustrate the appearance of experimental photoelastic fringe patterns, the benchmark problem of a ring under diametral compression (inner diameter = 40 mm, outer diameter = 80 mm, thickness = 6 mm, \( F_{\sigma} = 11 \) N/mm/fringe and load = 503 N) is chosen. The dark field isochromatics are viewed through a circular polariscope and recorded using a monochrome CCD camera (Sony XC-ST50) with sodium vapor (\( \lambda = 589.3 \) nm) light source and also using a 3CCD camera (Sony XC-003P) with white light source. The experimentally recorded dark field isochromatics are shown in grayscale and in colour in Figs. 1a and 1b, respectively.

**Procedure to Plot Photoelastic Fringes**

Plotting of line contours or arbitrarily coloured contours is quite common in the post-processing module of the commercially available FE packages. The focus in such plots is just to present the plotting variables of the problem as a contour. On the other hand, the fringe contours have varying thicknesses and colours and mimicking these aspects would help to compare the numerical results with the experimental results better. Plotting of contours is not only for validating a numerical model, but it can also be used to verify the experimental results in special cases [5].

**Computing the Whole Field Isochromatic Parameter**

The total fringe order, \( N \) is related to the principal stress difference by the famous stress-optic law [1,2]

\[
N = \frac{\left(\sigma_1 - \sigma_2\right)}{F_\sigma} h
\]

where, \( \sigma_1 \) is the maximum in-plane principal stress, \( \sigma_2 \) is the minimum in-plane principal stress, \( h \) is the specimen thickness, and \( F_\sigma \) is the material stress fringe value. Using the maximum and minimum in-plane stresses computed from the FE analysis for the entire model domain, the total fringe order is computed using Eq. (1).

**Definition of Spectrum for Contour Plots**

Commercial FE packages have several predefined contour spectrums which are used to colour the contour plots over the whole field. The most commonly used ones among these are the rainbow spectrum (used to plot stress and displacement fields) and the red to blue spectrum (used to plot temperature distribution). In order to simulate the photoelastic fringe contours realistically, new colour spectrums need to be defined. In Abaqus v6.7, colours are represented in the format ‘#rrggbb’ where \( rr \), \( gg \) and \( bb \) represent the hexadecimal values of the intensities of the red (R), green (G) and blue (B) planes. The decimal \( R \), \( G \) and \( B \) values range, in general, from 0 to 255 and their hexadecimal equivalents \( rr \), \( gg \) and \( bb \) range from 00 to FF.

For dark field isochromatics observed using a monochromatic light source experimentally, the intensity, \( I_d \) at any point is given by,

\[
I_d = 255 \times \sin^2 \left( \frac{\delta}{2} \right)
\]

where, \( \delta \) is the relative retardation defined by

\[
\delta = 2 \pi N
\]

In Eq. (3), \( N \) denotes the fringe order at the point of interest. The grayscale spectrum is thus defined by setting the \( R \), \( G \) and \( B \) intensities equal to \( I_d \). Table-1 gives the spectrum defined in steps of 0.1 fringe orders, which is found to be sufficient for most cases.

For plotting contours in colour, the \( R \), \( G \) and \( B \) intensity values are obtained from a colour code table. With developments in RGB photoelasticity [10] and three-fringe
photoelasticity [11], detailed colour code up to three fringe orders is available in the literature [11]. However, for the present use, the ‘#rrggbb’ values tabulated in Table-1 for fringe orders from 0 to 3 is found to suffice. For regions where the fringe order exceeds 3, the colours of the dark field isochromatics are known to merge. Therefore, for such regions, a constant colour of red is specified.

It can be seen from Table-1 that the grayscale spectrum follows a repetitive pattern which can be extrapolated to higher fringe orders using the same principle, but the colour spectrum is not to be extrapolated to higher fringe orders for reasons noted earlier. The spectrums are defined manually once and can be saved as an editable macro file for use in subsequent sessions. The continuous spectrum option is made use of for both the cases in order to obtain a smooth variation of contours across the entire model domain.

**Finite Element Simulation**

A 2D model of the ring is created to the same dimensions as the one used for experiment (Fig.1), and the FE analysis is done using ABAQUS v6.7. The model is discretized using 3648 eight-noded quadrilateral elements (Fig.2a). The boundary conditions and the diametral load of $503 \, N$ are applied to the FE model. The material properties of Araldite ($E = 3300 \, MPa$ and $\nu = 0.3$) are used for the analysis. The total fringe order is evaluated for the entire model domain using Eq. (1). The dark field isochromatics in grayscale and colour (Figs.2b and 2c) are plotted by selecting the respective spectrums defined in Table-1. For the colour plot, the limits are set as 0 to 3 and red colour is specified for zones which exceed this limit. For the grayscale plot, the limits are set as 0 to 5 and the grayscale spectrum is accordingly defined. It can be observed that the fringe patterns plotted from FE results (Fig.2b and 2c) compare very well with the experimental fringe patterns (Figs.1a and 1b).

**Application to a Coupled Dual-Beam Spring Element**

A photoelastic model (of thickness 6 mm) of a coupled dual-beam spring element (Fig.3a) is considered next to demonstrate the application of the plotting utility to a more complex problem. The experimental dark field isochromatics are shown in Figs.3b and 3c in grayscale and in colour, respectively.

The specimen is discretized using 1325 six-noded triangular elements (Fig.3a) to demonstrate that the pro-
posed methodology works with different element types. The FE results are then post processed to simulate the photoelastic fringes. The simulated dark field isochromatics given in grayscale (Fig.3d) and in colour (Fig.3e), compare well with the experimental results (Figs.3b and 3c).

Application to Transient Thermal Stress Problem

One of the major advantages of the proposed methodology is that its use in transient problems is simplified, as no input files need to be created for each time step as would be necessary if a separate post-processing software were to be used. Additionally, the dynamic rendering capability of the FE package may also be used to its fullest capacity. To demonstrate this, a transient thermal stress problem [9] is considered. An aluminium-polycarbonate bimaterial strip (of thickness 6 mm) with an interface crack (of length 18 mm) is prepared (Fig.4a) at an elevated temperature (36°C) and suddenly exposed to room temperature (28°C) where it gets cooled by convection heat loss to the surroundings. Transient dark-field isochromatics of the polycarbonate portion of the bimaterial strip are recorded and shown for a few time steps (Figs.4b-4e).

To simulate this experimental phenomenon, a transient, fully-coupled, temperature-displacement analysis is carried out according to the procedure given in [9]; the FE mesh is shown in Fig.4f. From the FE results, the dark field isochromatics are simulated using the proposed strategy for the same time steps, and shown in Figs.4g-4j. Though they predict slightly higher stresses overall, they compare quite well with the experimentally recorded isochromatics both in shape and in distribution of the fringe patterns (Figs.4b-4e).

Phasemaps in Digital Photoelasticity

Phase-shifting techniques are commonly used for whole-field quantitative estimation of the isochromatics (related to principal stress difference) and the isoclinics (principal stress direction). For visualization, these photoelastic parameters are represented as grayscale images called phasemaps. In isochromatic phasemaps, the variable is the fractional fringe order (0 < δ < 1) while for isoclinic phasemaps, the variable is the principal stress direction (-π/2 < θ < π/2). The direct trigonometric evaluation of the principal stress direction does not guarantee the evaluation of only one of the principal stress directions consistently over the domain. This results in the formation of zones in the experimental isoclinic phasemaps which represent the other principal stress direction, called as inconsistent zones. These zones lead to the formation of ambiguous zones in the isochromatic phasemap, wherein the fractional fringe order gradient direction is reversed [1, 2].

Experimental Photoelastic Phasemaps

To illustrate the appearance of photoelastic phasemaps, ten-step phase-shifted images [12] are recorded using a monochrome CCD camera (Sony XC-ST50) having a resolution of 768 x 576 pixels and a sodium vapor light source (λ = 589.3 nm). Experimentally, only the wrapped isoclinic value (with inconsistent zones) is obtained in the range -π/4 < θ < π/4; this is plotted as the wrapped isoclinic phasemap (Fig.5a). This needs to be unwrapped in the range -π/2 < θ < π/2 using an adaptive quality guided phase unwrapping algorithm [12] to obtain the isoclinic phasemap free of inconsistent zones. The resultant unwrapped isoclinics are then smoothed by an adaptive smoothing algorithm [13] using a span width of 25 pixels. Figs.5b and 5c show the experimentally obtained unwrapped isoclinic phasemap in grayscale and as a binary plot, respectively. Following the ten-step method, the unwrapped isoclinic data is used to evaluate the isochromatics which are free of ambiguity; the experimentally obtained isochromatic phasemap is given in Fig.5d.

Procedure to Plot Photoelastic Phasemaps

To perform unwrapping of isoclinic values, a seed point needs to be selected in one of the correct zones, for which the correct and inconsistent zones need to be identified first. Identifying all the inconsistent zones from the wrapped isoclinic phasemap could be tedious for complex problems, especially when many isotropic lines / pi jumps [12, 13] are present. Often, for complex industrial problems, both experimental and numerical studies are carried out and the finite element (FE) method could be of help in such situations to identify the inconsistent zones. The actual values can still be evaluated experimentally and compared with FE for its validation.

In the sub-sections that follow, the methodology for plotting isoclinic and isochromatic phasemaps is presented.
Simulation of Isoclinic Phasemaps

The principal stress direction can be computed for the whole-field using the relation

\[
\theta = \frac{1}{2} \tan^{-1} \left( \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \right)
\]

where, \(\tau_{xy}\), \(\sigma_x\) and \(\sigma_y\) are the shear and normal stresses. When the atan function is used, the wrapped isoclinic value (with inconsistent zones) in the range \(-\pi/4 < \theta < \pi/4\) is obtained. When the atan2 function is used in Eq. (4), the continuous isoclinic values in the range \(-\pi/2 < \theta < \pi/2\) are obtained. It is thus possible to directly obtain the unwrapped isoclinic value from FE results as one knows the correct sign of the numerator and denominator independently; in experiments, this luxury is not available since only intensity information is processed. Since the implementation of the atan2 function is not available directly in Abaqus v6.7, it needs to be defined separately (Appendix-A).

A colour spectrum with only black and white is defined and the continuous plotting option is specified to obtain a seamless variation of principal stress direction across the model domain. The limits are set as \(-\pi/2\) and \(\pi/2\), and one obtains the wrapped isoclinic phasemap when atan function is used and the unwrapped isoclinic phasemap when the atan2 function is used. Figs.6a and 6b show the wrapped and unwrapped isoclinic phasemap for the problem of a ring under diametral compression, respectively.

It is also possible to plot the unwrapped isoclinic as a binary plot which is very useful to clearly visualize the isoclinic lines quantitatively. The absolute values of the unwrapped isoclinics range from 0° to 90°. The binary plot is of banded nature with a spectrum of alternating white and black colours. Each band is of 5° width (that is ±2.5°) and the black bands are centered at isoclinics in multiples of 10°. Thus, a spectrum is defined with 9 pairs of alternating black and white colours and the absolute value of unwrapped isoclinic in degrees is plotted as a banded contour with the number of bands set to 18 and the limits specified as 2.5 and 92.5, in order to get the isoclinics in steps of 10°. Below the specified limit, black colour is specified. The binary representation of the simulated unwrapped isoclinic phasemap is given in Fig.6c. Table-2 gives a summary of the colour spectrums and limits used to simulate the phasemaps.

Simulation of Isochromatic Phasemap

Experimentally using the ten-step method [12], the unwrapped isoclinics are used to evaluate the isochromatic phasemap free of ambiguity. In order to simulate this from FE results, the total fringe order is computed for the whole field using Eq. (1). It is to be noted that one does not make use of the principal stress direction / isoclinic data at all in its computation, making it fairly straightforward to plot isochromatic phasemaps without the presence of ambiguous zones.

A colour spectrum needs to be defined as indicated in Table-2, which is essentially a gradual increase in gray level intensity from black to white, followed by a sudden fall at whole fringe orders. The table can be suitably extrapolated following the same sequence for plotting higher fringe orders as well. The limits are set as 0 and \(N_{\text{maxdef}}\), where \(N_{\text{maxdef}}\) is the number of fringe orders that has been defined in the colour spectrum. The continuous plotting option is used and the resultant plot is the isochromatic phasemap without ambiguous zones. For the problem of a ring under diametral compression, the isochromatic phasemap without ambiguous zones is plotted in Fig. 6d and is seen to compare well with its experimental counterpart (Fig.5d).

Application to a Coupled Dual Beam Spring Element

For the coupled dual-beam spring element (Fig.3) described earlier, Figs.7a, 7b and 7c show the wrapped and unwrapped isoclinic phasemaps plotted from FE results; the isochromatic phasemap is shown in Fig.7d. The unwrapped isochromatic phasemap plotted from FE results (Fig.7b) reveals the presence of isotropic points and their associated pi jumps. The knowledge of the pi jumps a priori will be helpful for the experimentalist in processing the experimentally obtained wrapped isoclinic phasemap (Fig.7e). Using the isochromatic phasemaps plotted from FE results, a suitable seed point is selected for unwrapping. The experimental isoclinic values are unwrapped and smoothed and plotted as phasemaps using the grayscale and binary representations in Figs.7f and 7g, respectively. The experimental isochromatic phasemap free of ambiguous zones is shown in Fig.7h. Comparing the experimental phasemaps (Figs.7e-7h) with the phasemaps plotted from FE results, (Figs.7a-7d), it is clear that the correlation is very good.
Conclusion

A simple method has been developed to plot photoelastic fringe contours and phasemaps, both isoclinic and isochromatic, directly from a commercial FE package (Abaqus v6.7), using only the standard post-processing features available therein. Such a direct simulation removes the constraints placed on the mesh or type of element used for the FE analysis, as against the case when a specially developed post-processing software is used. In solving transient problems, post-processing each time step separately would be very cumbersome; in this context, directly using the available features for photoelastic simulation is shown to be very advantageous. In addition, plotting of phasemaps enables identification of inconsistent zones in isoclinic phasemaps, especially for complex problems, in order to select a seed point for unwrapping.

Table-2: Definition of Colour Spectrums Used for Simulating Photoelastic Phasemaps

<table>
<thead>
<tr>
<th>Isoclinic Phasemap</th>
<th>Isochromatic Phasemap</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grayscale</strong></td>
<td><strong>Binary plot</strong></td>
</tr>
<tr>
<td>Theta</td>
<td>Absolute Theta</td>
</tr>
<tr>
<td>90</td>
<td>'#FFFFFF'</td>
</tr>
<tr>
<td>90</td>
<td>'#000000'</td>
</tr>
<tr>
<td>-90</td>
<td>'#000000'</td>
</tr>
<tr>
<td>90</td>
<td>'#FFFFFF'</td>
</tr>
</tbody>
</table>

Lower Limit

<table>
<thead>
<tr>
<th>Upper Limit</th>
<th>Colour below lower limit</th>
<th>Colour above upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>-90</td>
<td>'#000000'</td>
<td>'#FFFFFF'</td>
</tr>
<tr>
<td>90</td>
<td>'#FFFFFF'</td>
<td>'#000000'</td>
</tr>
</tbody>
</table>

Spectrum Type

<table>
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<th>Continuous</th>
<th>Banded (18)</th>
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</thead>
</table>

Problem Specific

<table>
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<tr>
<th>Fringe Order</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>'#000000'</td>
</tr>
</tbody>
</table>

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179
This will help the photoelasticians to solve complex problems.

References


Appendix-A

The atan function does not distinguish between diametrically opposite directions for the angle $\beta$, and gives the result in the range $-\pi/2 < \beta < \pi/2$. The atan2 function takes into account the signs of the numerator and denominator and places the angle in the correct quadrant in the range $-\pi < \beta < \pi$. In terms of the atan function, the atan2 function can be expressed as follows:

$$\text{atan2} \left( \frac{y}{x} \right) = \begin{cases} \text{atan} \left( \frac{y}{x} \right) & \text{if } x > 0 \\ \pi + \text{atan} \left( \frac{y}{x} \right) & \text{if } x < 0 \text{ and } y > 0 \\ -\pi + \text{atan} \left( \frac{y}{x} \right) & \text{if } x < 0 \text{ and } y < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0 \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0 \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0 \end{cases}$$

Since the 'if ... else ... loop' is also not defined in Abaqus v6.7, the atan2 function needs to be explicitly defined as

$$\text{atan2} \left( \frac{y}{x} \right) = \left[ \frac{\pi}{2} \times \text{sgn} \left( y \right) \left( 1 - \text{sgn} \left( x \right) \right) \right] + \left[ \text{sgn} \left( x \right) \times \text{atan} \left( \frac{y}{x} \right) \right]$$
Fig. 1 Experimental Dark Field Isochromatics of Ring Under Diametrical Compression (a) in Grayscale (b) in Colour

Fig. 2 Ring Under Diametral Compression (a) FE Mesh. Dark Field Isochromatics Plotted from FE Results (b) in Grayscale (c) in Colour

Fig. 3 Coupled Dual-beam Spring Element (a) FE Mesh. (b) Experimental Dark Field Isochromatics (b) in Grayscale (c) in Colour. Dark Field Isochromatics Plotted from FE Results (d) in Grayscale (e) in Colour

Fig. 4 Aluminium-Polycarbonate Bimaterial Specimen with Centre Crack Cooled by Convection from a Bonding Temperature of 36°C to 28°C. (a) Specimen used for Experiment (b-e) Experimental Transient Isochromatic Fringe Patterns of Polycarbonate Portion Results (f) FE Mesh (g-i) Transient Isochromatic Fringe Patterns Plotted from FE Results

Fig. 5 Ring Under Diametral Compression. Experimental Results: (a) Wrapped Isoclinic Phasemap (b) Unwrapped Isoclinic Phasemap (c) Binary Plot of Isoclinic Phasemap (d) Wrapped Isochromatic Phasemap
Fig. 6 Ring Under Diametral Compression. Phasemaps Plotted from FE Results: (a) Wrapped Isoclinic Phasemap (b) Unwrapped Isoclinic Phasemap (c) Binary Plot of Isoclinic Phasemap (d) Wrapped Isochromatic Phasemap

Fig. 7 Coupled Dual-beam Spring Element. Phasemaps Plotted from FE Results: (a) Wrapped Isoclinic Phasemap (b) Unwrapped Isoclinic Phasemap (c) Binary Plot of Isoclinic Phasemap (d) Wrapped Isochromatic Phasemap. Experimental Results: (e) Wrapped Isoclinic Phasemap (f) Unwrapped Isoclinic Phasemap (g) Binary Plot of Isoclinic Phasemap (h) Wrapped Isochromatic Phasemap