GENERALIZATION OF MVCCI APPROACH FOR LEFM PROBLEMS USING NUMERICAL INTEGRATION

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Abstract

Modified Virtual Crack Closure Integral (MVCCI) has become a valuable tool for estimation of mixed-mode fracture parameters in Linear Elastic Fracture Mechanics (LEFM) problems. When finite elements with fewer nodes and using polynomial shape functions are employed at the crack tip for fracture analysis, the expressions for MVCCI can easily be derived and many times also be expressed in closed form. For higher order elements they become unwieldy and so a numerical integration scheme is proposed and this can be used with any type of element. The scheme requires a two-step numerical integration: one to evaluate nodal forces exerted by the solid below the crack extension line on the portion above and the second to evaluate Strain Energy Release Rates. This development makes the post-processing for fracture parameters easy and computationally economical for 3-dimensional fracture problems. The technique is used to estimate fracture parameters in certain practical problems using higher order elements and typical numerical results are presented.

Introduction

The fracture behavior of many practical structures can be predicted on the principles of Linear Elastic Fracture Mechanics [1] including failure under overloads as well as fatigue crack growth leading to failure. Stress Intensity Factors (SIF-K) and Strain Energy Release Rates (SERR-G) in the three modes of fracture are the parameters to be determined based on LEFM. Modified Virtual Crack Closure Integral (MVCCI) has been developed over years as a very effective technique for this purpose and has been used for fracture analysis for several problems. The technique is simple and capable of resolving the fracture parameter components in individual modes in mixed-mode problems. Typical references are [2-6].

MVCCI was based on the Crack Closure Integral proposed by Irwin [7] which states that the strain energy release rate during a virtual crack extension is equal to the work required to close the crack back to the original size. The approach for this was contemplated as a two step analysis of bodies with marginally differing crack sizes and SERR was estimated as the difference in strain energy between these two configurations. Numerical difficulties are expected in such an exercise. Rybicki and Kanninen [2] were the first to propose the modified crack closure integral from a single finite element analysis and estimate SERR components from the nodal forces and crack face displacements in the vicinity of the crack tip. Here it is assumed that the crack tip elements are very small and represent the assumed virtual crack extension. It was realized that the equations presented for MVCCI are dependent on the type of element used at the crack tip. Original equations of Ref. [2] are only valid for 3-node triangular or 4-node quadrilateral QUAD4 elements. Buchholz [3] extended the approach to 6-node triangular and 8-node quadrilateral QUAD8 elements. Subsequently the technique has been generalized and made applicable for several regular and singular elements [4-6].

In spite of all the above mentioned developments still the problem exists that one need to use element dependent equations for MVCCI calculations. In the recent work by the authors and their colleagues, a numerically integrated NI-MVCCI [8-9] was proposed which overcomes many of the concerns in the use of MVCCI. These have been successfully used for the problems of plates/ stiffened panels under extension, bending and shear loads [9]. Further, the benefits of NI-MVCCI are obvious in 3-dimensional crack problems. Though it is possible to derive MVCCI equations in closed form for special cases of cubic

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HEXA8 elements at the crack tip, these equations become unwieldy for the cases of HEXA20 or HEXA27 brick elements along the crack front. NI-MVCCI is the natural choice for such cases. The major point to realize in this case is that it requires a two step numerical integration: one for estimating the nodal forces and the second for estimating components of SERR. This paper presents a consolidated view of this approach and present typical numerical results. Results from studies on cracks in stiffened rectangular panels under uni-axial tension are presented with 2-dimensional analysis and on de-lamination at various interfaces in laminated composite panels under compression are presented with 3-dimensional analysis.

Two-Dimensional Fields with Cracks.

Modified Crack Closure Integral can be expressed (Fig.1) to evaluate SERR in Mode-I and Mode-II for 2-dimensional problems as follows:

\[
G_I = \lim_{\Delta a \to 0} \frac{1}{2 \Delta a} \int_0^{\Delta a} \sigma_z (r, \Theta = 0) w (\Delta a - r, \Theta = \pi) bda
\]

\[
G_{II} = \lim_{\Delta a \to 0} \frac{1}{2 \Delta a} \int_0^{\Delta a} \tau_{xz} (r, \Theta = 0) u (\Delta a - r, \Theta = \pi) bda
\]

(1)

Following Ref.[2, 4] the above integral can be expressed in terms of the nodal forces and nodal displacements as shown in Fig.2 for QUAD4 and QUAD 8 elements as follows:

\[
G_I = \frac{1}{2 \Delta a} F_z w
\]

\[
G_{II} = \frac{1}{2 \Delta a} F_x u
\]

(2)

In both the cases, \( G_{Total} = G_I + G_{II} \)

The above equations can be derived by assuming polynomial distributions for stresses and displacements in the integrals in Eq.(1) and integrating over the length of virtual crack extension [6]. This will be described for the 3-dimensional case later. In the numerical computation, the convergence is studied by progressively decreasing the crack tip element size (implying virtual crack extension) to a small value in evaluating the integrals given in Eq.(1).

Three-Dimensional HEXA Elements

The virtual crack front extension for 3-dimensional problems is shown in Fig.3. The stresses normal to the crack plane and the crack opening displacements can be expressed in polynomials for HEXA8 element in well known natural co-ordinate system as

\[
\sigma_z (\xi, \eta) = b_0 + b_1 \xi + b_2 \eta + b_3 \xi \eta + b_4 \xi^2 + b_5 \eta^2 + b_6 \xi^2 \eta + b_7 \eta^2 \xi^2
\]

\[
w (\xi, \eta) = a_0 + a_1 \xi + a_2 \eta + a_3 \xi \eta + a_4 \xi^2 + a_5 \eta^2 + a_6 \xi^2 \eta + a_7 \eta^2 \xi^2
\]

(3)

By using the relation between the displacement at any point in the element to the nodal displacements given by

\[
w = N_i w_i
\]

(4)

By substituting the nodal coordinates and the nodal displacements in Eq.(4) it is possible to derive the coefficients in the displacement function of Eq. (3) as

\[
\begin{bmatrix}
N_0 & N_1 & \cdots & N_7
\end{bmatrix}
\begin{bmatrix}
N_1 & N_1 & \cdots & N_1
N_1 & N_1 & \cdots & N_1
N_1 & N_1 & \cdots & N_1
N_1 & N_1 & \cdots & N_1
\end{bmatrix}^{-1}
\begin{bmatrix}
w_1
w_2
w_3
\end{bmatrix}
\]

(5)

The nodal forces are expressible in terms of the stresses and shape functions as an integral given by

\[
F_{z,i} = \int_{-1}^{1} \int_{-1}^{1} \frac{1}{|J|} \sigma \cdot d \xi d \eta
\]

(6)

Substituting for stresses from Eq.(3), one gets
These nodal forces are the forces exerted by the solid below the crack extension line on the portion above the crack extension line. These are shown for HEXA8 element at the crack front in Fig.4. The nodes involved on crack extension line are shown in Fig.5 for HEXA20 element at the crack front. The above integration can be carried out in closed form and the coefficients in stress distribution in Eq. (3) can be expressed as

\[
\begin{bmatrix}
  b_0 \\
  b_1 \\
  b_2 \\
  b_3 \\
  b_4 \\
  b_5 \\
  b_6 \\
  b_7
\end{bmatrix} = \int_{-1}^{1} \int_{-1}^{1} \left[ N_j \right]^T \begin{bmatrix} 1 & \xi & \eta & \xi^2 & \eta^2 & \xi \eta & \xi^2 \eta & \xi \eta^2 \end{bmatrix} |J| d\xi d\eta \tag{7}
\]

\[
\begin{bmatrix}
  b_0 \\
  b_1 \\
  b_2 \\
  b_3 \\
  b_4 \\
  b_5 \\
  b_6 \\
  b_7
\end{bmatrix} = \begin{bmatrix}
  N_1 & N_4 \xi & N_7 \eta & N_{10} \xi \eta & N_2 \xi^2 & N_5 \eta^2 & N_{11} \xi \eta^2 \\
  N_2 & N_5 \xi & N_8 \eta & N_{11} \xi \eta & N_3 \xi^2 & N_6 \eta^2 & N_{12} \xi \eta^2 \\
  N_3 & N_6 \xi & N_9 \eta & N_{12} \xi \eta & N_4 \xi^2 & N_7 \eta^2 & N_{13} \xi \eta^2 \\
  N_4 & N_7 \xi & N_{10} \eta & N_{13} \xi \eta & N_5 \xi^2 & N_8 \eta^2 & N_{14} \xi \eta^2 \\
  N_5 & N_8 \xi & N_{11} \eta & N_{14} \xi \eta & N_6 \xi^2 & N_9 \eta^2 & N_{15} \xi \eta^2 \\
  N_6 & N_9 \xi & N_{12} \eta & N_{15} \xi \eta & N_7 \xi^2 & N_{10} \eta^2 & N_{16} \xi \eta^2 \\
  N_7 & N_{10} \xi & N_{13} \eta & N_{16} \xi \eta & N_8 \xi^2 & N_{11} \eta^2 & N_{17} \xi \eta^2 \\
  N_8 & N_{11} \xi & N_{14} \eta & N_{17} \xi \eta & N_9 \xi^2 & N_{12} \eta^2 & N_{18} \xi \eta^2 \\
  N_9 & N_{12} \xi & N_{15} \eta & N_{18} \xi \eta & N_{10} \xi^2 & N_{13} \eta^2 & N_{19} \xi \eta^2
\end{bmatrix}^{-1}
\begin{bmatrix}
  F_{z,j} \\
  F_{z,j+1} \\
  F_{z,j+2} \\
  F_{z,j+3} \\
  F_{z,j+4} \\
  F_{z,j+5} \\
  F_{z,j+6} \\
  F_{z,j+7}
\end{bmatrix} \tag{8}
\]

where the Jacobian is given as

\[
|J| = \begin{bmatrix}
  \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \eta} \\
  \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \eta}
\end{bmatrix} = J_0 + J_1 \xi + J_2 \eta + J_3 \xi^2 + J_4 \xi \eta + J_5 \eta^2 + J_6 \xi^3 + J_7 \xi^2 \eta + J_8 \xi \eta^2 + J_9 \eta^3 + J_{10} \xi^2 \eta^2 \tag{9}
\]
The final integration for SERR components can be written as

\[
(a_0 + a_1 \xi + a_2 \eta + a_3 \xi \eta + a_4 \xi^2 + a_5 \eta^2 + a_6 \xi^2 \eta + a_7 \xi \eta^2)
\]

\[
G_I = \frac{1}{2 \Delta A} \int_{-1}^{1} \int_{-1}^{1} \left( b_0 + b_1 \xi + b_2 \eta + b_3 \xi \eta + b_4 \xi^2 + b_5 \eta^2 + b_6 \xi^2 \eta + b_7 \xi \eta^2 \right) \left( J_0 + J_1 \xi + J_2 \eta + J_3 \xi \eta + J_4 \xi^2 + J_5 \eta^2 + J_6 \xi^2 \eta + J_7 \xi \eta^2 \right) \, d\xi \, d\eta
\]

(10)

Where \( A_k \) is the average of the areas of the elements ahead and behind the crack front. The expressions for the mode-II and mode-III components can be written on similar lines. On the other hand for HEXA20 and HEXA27 elements the integral for coefficients in the stresses are unwieldy to evaluate in closed form and can conveniently be obtained by numerical integration. The full integral can be expressed as given in Eq. 11 given below and this integral for Strain Energy Release Rate components can also be carried out by numerical integration.

\[
(a_0 + a_1 \xi + a_2 \eta + a_3 \xi \eta + a_4 \xi^2 + a_5 \eta^2 + a_6 \xi^2 \eta + a_7 \xi \eta^2 + a_8 \xi^2 \eta^2)
\]

\[
(b_0 + b_1 \xi + b_2 \eta + b_3 \xi \eta + b_4 \xi^2 + b_5 \eta^2 + b_6 \xi^2 \eta + b_7 \xi \eta^2)
\]

\[
G_I = \frac{1}{2 \Delta A} \int_{-1}^{1} \int_{-1}^{1} \left( J_0 + J_1 \xi + J_2 \eta + J_3 \xi \eta + J_4 \xi^2 + J_5 \eta^2 + J_6 \xi^2 \eta + J_7 \xi \eta^2 + J_8 \xi^2 \eta^2 + J_9 \eta^3 + J_{10} \xi^2 \eta^2 \right) \left( J_0 + J_1 \xi + J_2 \eta + J_3 \xi \eta + J_4 \xi^2 + J_5 \eta^2 + J_6 \xi^2 \eta + J_7 \xi \eta^2 + J_8 \xi^2 \eta^2 + J_9 \eta^3 + J_{10} \xi^2 \eta^2 \right) \, d\xi \, d\eta
\]

(11)

Here too the expressions for mode-II and mode-III components can be written on similar lines. Gaussian numerical integration was carried out on the integrals in Eqs. (6) and (11). In 3-dimensional problems the stress intensity factors are obtained assuming plane strain relation between \( G \) and \( K \).
Software and its Capability

The numerical studies in the current paper were carried out with software DGAMNAS developed for analysis of Aerospace structural components covering metallic and composite structures. Linear, geometrically non-linear and material non-linear analysis can be handled with this software. The current analysis used geometric non-linear capability for problems such as large deformations of cantilever and de-lamination studies of laminated composites under compression. For composites, special super elements are developed with more than one layer within a single element. The capability to estimate fracture parameters using MVCCI technique is embedded into this software. The solid modeling and the finite element meshes were developed from PATRAN and interface routines are developed to translate the input file to DGAMNAS. Special in-house routines are developed to mesh circular or elliptical crack/de-lamination using conformal mapping methods.

Numerical Results and Discussion

Numerically Integrated MVCCI is applied to a series of problems with the aim to make this a feasible method for the failure analysis of structures in the presence of cracks. However the shape functions are in polynomial form, in principle, the integration can be performed exactly using an appropriate order of numerical integration. In case of singular elements where the shape functions are singular terms besides polynomial terms, it is necessary to identify the order of integration required to achieve desired accuracy in estimating SERR components. So, the convergence of the integration scheme was tested with several orders of integration. The major benefit in the use of MVCCI will be seen for 3-dimensional part-through cracks. The principle of using MVCCI to determine SIF variation along the crack front is explained in the previous sections. Some typical numerical results are presented in Figs. 6-12 and these are discussed here.

Cantilever Under Large Deformation

The first exercise was to try the current software on a typical problem without crack. For this purpose the well known problem of cantilever under tip load with tip displacements in the range of large deformation is studied with three types of 3-D elements HEXA8, HEXA20, HEXA27. The Fig.6 also shows linear solution, Elastica [12], and FEM computations with the three elements. It is clearly seen that both HEXA20 and HEXA27 are much better than HEXA8. The eight node brick element is known to have difficulty in dealing with problems involving bending. Among the two higher order elements, HEXA27 is closer to ELASTICA. This is interesting since HEXA27 is the most neglected element in literature. However, further work could not be carried out with HEXA27 since the standard pre-processors do not have the capability to create meshes with HEXA27 element. All further work was carried out with HEXA20 elements.

Double Cantilever Beam

The Double Cantilever Beam (Fig.7) is one of the standard specimens for mode-I testing. The Double Cantilever Beam is modeled with all the three brick elements and the stress intensity factors estimated at the centre of the specimen are shown in Table-1. The results are compared with the solution provided in Ref. [13]. The table shows that FE analysis with all the three elements generated accurate results within 1.2% to the reference solution.

Penny Shaped Crack

The next case studied is the penny shaped crack in a cylindrical solid subjected to remote tension. The problem is axi-symmetric and so only a quarter of the domain was modeled with HEXA8 elements. The stress intensity factor along the crack front (which is theoretically constant) is shown in Fig.8 along with the plane strain solution which should be exact, provided the crack radius is small compared to the radius of the cylindrical solid. The solution is seen to be converging with increasing number of FEM mesh size.

Two Dimensional Stiffened Panel

The current method was successfully applied for standard problems where near exact solutions are available in literature. These include mode-I and mode-II loading on crack in a large plate and SENT specimens. Further, known to have difficulty in dealing with problems involving bending. Among the two higher order elements, HEXA27 is closer to ELASTICA. This is interesting since HEXA27 is the most neglected element in literature. However, further work could not be carried out with HEXA27 since the standard pre-processors do not have the capability to create meshes with HEXA27 element. All further work was carried out with HEXA20 elements.

| Table-1 : SERR (Gj) for DCB specimen : Comparison of results with Hexa8, Hexa20 and Hexa 27 elements |
|-----------------|-----------------|----------------|
| Element Type    | MVCCI Gj J/m²  | % Deviation*   |
| Hexa 8          | 1.915E+03       | 0.26           |
| Hexa20          | 1.932E+03       | 0.625          |
| Hexa27          | 1.943E+03       | 1.19           |

* Deviation with reference to [13] Kanninen et al., \( G_I = 1.92E+03 \) J/m².
a practical 2-dimensional stiffened panel problem is studied and is presented in this paper along with comparison with existing results. The finite element mesh using 4-node plate bending elements for this problem is shown in Fig.9. The stiffeners are represented by beam elements. The current results with both eccentric and concentric locations of the stiffener are presented in Fig.10. The results with concentric location of the stiffeners from Rooke and Cartwright [10] are also shown for comparison. Here As is the area of stiffener and Ap is the sectional area of the plate. The stress intensity factors compare well and they deviate only when the stiffener area is very large. Large stiffener area is likely to result in not well represented idealization in 2-dimensional analysis.

Experimental Comparison

While establishing a method, it is necessary that comparisons are made with results obtained from experiments. A delaminated laminated composite panel was subjected to compression loads and the lateral displacement at the centre of the panel was measured [14]. This is shown along with the current simulation of this problem by 3-dimensional FEM with HEXA8 and HEXA20. These are shown in Fig.11. Initially the delaminated sub-laminate buckles and this is seen by the deformation which progressively increases with increasing load level measured by compressive strain on the edges of the panel. At loads beyond certain level, the overall laminate buckles and the lateral displacement at the centre of the panel raise rapidly. This deformation pattern was generated by FE analysis with the present software. Geometrically non-linear analysis was carried out due to large deformations expected at the centre of the panel. The FE analysis reproduced the results till the buckling of the overall panel which can not be obtained in static analysis.

De-lamination Tolerance

Finally an exercise to define de-lamination tolerance in laminated panels is generated from the present work [15] and this is presented in Fig.12. The maximum mode-I strain energy release rate along the de-lamination front is plotted versus compressive strain on the panel for de-lamination of various sizes. The limit load level is fixed at 4000 micro-strains. The SERR is compared with threshold value to determine the tolerance level. It can be seen from the figure that more than 50 mm diameter de-lamination could cause unstable growth under fatigue loading.

Concluding Remarks

The NI-MVCCI has been successfully used for several problems within the linear and geometrically non-linear deformations. The results from the present analysis are successfully compared with those available in literature. The technique is used to determine strain energy release rate level at the crack/de-lamination fronts. For problems of de-lamination, this technique is used to define tolerance levels for given limit load levels. Numerical integration procedure eliminates some of the drawbacks of basic MVCCI technique, where different set of equations need to be used for different types of elements at the crack tip. The technique is extremely useful in 3-dimensional problems involving determination of fracture parameters at crack/de-lamination fronts.

References


Fig. 4 Nodal forces and displacements for 8-node brick elements

Fig. 5 Nodes of the elements behind and ahead of the crack front in 3-d problems: 20-node brick

Fig. 6 Geometrically non-linear analysis of cantilever beam with Hexa8, Hexa20 and Hexa27 elements [Ref. 12]
Fig. 7 Double cantilever beam (DCB) configuration

Fig. 8 Variation of $G_i$ along the embedded penny shaped crack front at 1000 $\mu$s.
Fig. 9 FE idealization of the rectangular stiffened plate (quarter symmetry)

Fig. 10 Variation of $K_I$ with respect to $S_a$ for $q/W = 0.2$ [Ref. 10, 11]
Fig. 11 Comparison of deflection versus load from various analyses and experimental data [Ref. 14]

Fig. 12 Variation of $G_i$ with applied compressive strain for different 'a' [Ref. 15]