DEVELOPMENT OF MATHEMATICAL MODEL OF A GENERIC RLV
FOR REENTRY MISSION SIMULATION

Ashok Joshi* and K. Sivan*

Abstract

This paper presents the nonlinear models for simulating the atmospheric reentry dynamics of a generic Reusable Launch Vehicle (RLV). The six degrees-of-freedom flight dynamics is modeled with the overall objective of minimum number of constraints without increasing the complexity of equations. This requirement leads to the definition and use of a number of frames of reference for describing different aspects of the dynamics of the RLV. In this context, the inertial reference frame is used for translation motion, while body frame and wind frame are used to describe the forces acting on the RLV. Further, quaternion representation is used for vehicle attitude motion in order to avoid computational singularities in respective kinematic model equations. The vehicle complex aerodynamic characteristics are modeled using the coefficient approach in order to simplify interpretation of available data while the propulsion system model takes into account the atmospheric conditions. The vehicle subsystem models of sensors, navigation system and actuators are also included. The operating environment models viz., Earth’s atmosphere, Earth’s shape and gravitational acceleration are also modeled appropriately. The model presented is for a generic RLV and has the potential for application to RLVs of widely different configurations.

Nomenclature

\[ A_{ei} = \text{exit area of } i\text{th engine} \]
\[ C_{Ax}, C_{Ny}, C_{Y} = \text{axial, normal and side force coefficients respectively} \]
\[ C_{r}, C_{m}, C_{n} = \text{rolling, pitching and yawing moment coefficients respectively} \]
\[ C_{N\alpha}, C_{Nq} = \text{normal force coefficient due to angle of attack and pitch rates} \]
\[ [E], \bar{e} = \text{quaternion matrix; quaternion} \]
\[ = [e_0, e_1, e_2, e_3]^T \text{ parameters} \]
\[ F_A, F_T = \text{aerodynamic and thrust force vectors in body frame} \]
\[ F_{Ti} = \text{thrust force vector of } i\text{th engine in body frame} \]
\[ \overline{G}_I = [G_X, G_Y, G_Z]^T = \text{gravity acceleration in ECI frame} \]
\[ h = \text{altitude} \]
\[ [H] = \text{inertia matrix} \]
\[ [IB] = \text{ECI frame to body frame transformation matrix} \]
\[ K_H, K_N = \text{weighting coefficients for control surface inputs} \]
\[ M = \text{Mach number} \]

\[ = \text{aerodynamic and thrust related moment vectors in body frame} \]
\[ = \text{moment due to thrust about body axes due to } i\text{th engine} \]
\[ = \text{vehicle mass} \]
\[ = \text{number of engines} \]
\[ p_a = \text{atmospheric pressure} \]
\[ p_{ei} = \text{exit pressure of } i\text{th engine} \]
\[ q_{dyn} = \text{dynamic pressure} \]
\[ = \text{position vector in ECI frame} \]
\[ = \text{position vector in body fixed frame} \]
\[ = \text{thrust location of } i\text{th engine} \]
\[ = \text{atmospheric/vacuum thrust of } i\text{th engine} \]
\[ = \text{relative velocity magnitude} \]
\[ = \text{inertial velocity vector in ECI frame} \]
\[ = \text{angles of attack and side slip; pitch rate} \]
\[ = \text{equivalent aileron/elevator deflections} \]
\[ = \text{control surface deflection commanded/actual} \]

* Department of Aerospace Engineering, Indian Institute of Technology Bombay, Powai, Mumbai-400 076, India
Email : ashokj@aero.iitb.ac.in
Manuscript received on 08 Nov 2004; Paper reviewed, revised and accepted on 22 Aug 2005
Introduction

Reusable Launch Vehicles (RLVs) are new generation space transportation systems that lift off vertically like a rocket or take off horizontally like an aircraft and land precisely either in vertical mode or in horizontal mode like an aircraft, refuel and get ready for next flight [1, 2]. While, the launch phase of RLV is similar to that of a conventional launch vehicle, the return flight from space to Earth consists of Keplerian atmospheric entry and landing phases. Among these, the atmospheric reentry phase is more complicated as this phase is expected to dissipate large amount of kinetic and potential energy. In addition, this phase is characterized by segments of very different environmental conditions and therefore a realistic simulation of such a flight trajectory, during the RLV design process, requires mathematical models that are capable of representing these different segments with good fidelity [2]. This paper presents the details of the nonlinear dynamic modeling of a generic RLV during its reentry phase and discusses the various effects that are considered important.

Reentry Dynamic Equations

Figure 1 shows the configuration of a generic RLV, visualized as a winged body, with control moments generated by control surfaces and a Reaction Control System (RCS).

Reentry dynamics of RLV consists of three translational and three rotational equations of motions, which contains the kinematic and dynamic terms. For the reentry flight, there are three kinds of external forces and moments acting on the vehicle. These are; the aerodynamic forces and moments, forces and moments due to propulsion system and gravitational forces. Further, the choice of coordinate system is of prime importance in representing reentry dynamics, as it influences the generality and versatility of the model. In the present paper, the modeling philosophy is to have minimum number of constraints, without increasing the complexity of equations. This has resulted in a number of different coordinate systems for different components of the mathematical model, in order to simplify component model descriptions [3]. However, this approach requires appropriate coordinate transformations to exchange the data and information between them (see Appendix for coordinate systems). Translational equations of motion are expressed in Earth Centered Inertial (ECI) frame, to avoid computational singularities and errors due to ill conditioning of matrices [3,4] and are given below as:

\[ \ddot{r} = \nabla_I \]
\[ \nabla_I = \left(I/m \right) \left[ \left[I \times \dot{\omega} \right]^{-1} \left[ F_A + F_T \right] + G_I \right] \]

The flight path, in terms of altitude, latitude, longitude, velocity magnitude and its orientation angles, is derived from the outputs expressed in ECI frame. The dynamic equations for angular motion are obtained from the Euler’s Law [4] and quaternians are used to represent attitude angles, to avoid singularity. These equations are as given below [3].

\[ \ddot{\omega} = \left[I \times \dot{\omega} \right]^{-1} \left[ \left[ M_A + M_T \right] - \left[I \times \dot{\omega} \right] \right] \]
\[ \dddot{\omega} = \frac{1}{2} \left[E \right] \ddot{\omega} \]
Vehicle Force Models

Aerodynamic forces, due to flow angles of attack and propulsive as well as aerodynamic control forces that are generated during its complete flight history, influence the RLV dynamics. The modeling for these is presented in following sub-sections.

Propulsion Model

The generic RLV uses rocket engine for de-orbiting manoeuvre and a RCS is used for vehicle control until aerodynamic control through control surface deflection becomes effective [3]. The net atmospheric thrust of ith engine along the nozzle axis is given by:

\[ T_i = T_{vi} + (P_{ei} - p_a)A_{ei} \]  

(5)

The orientation of thrust of ith engine with respect to body frame is defined by azimuth of thrust direction with respect to Y_B axis (ξ_i) and elevation angle of thrust direction with respect to X_B (η_i). The thrust force of ith engine along body axes is given by the equation.

\[ \bar{F}_{Ti} = T_i \begin{bmatrix} C_{\eta_i} \\ S_{\eta_i} C_{\xi_i} \\ S_{\eta_i} S_{\xi_i} \end{bmatrix} \]  

(6)

Moment due to thrust force of ith engine about body axes is given by the equation,

\[ \bar{M}_{Ti} = r_{Ti} \times \bar{F}_{Ti} \]  

(7)

The total thrust force along body axes and thrust moment about body axes are given by:

\[ \bar{F}_T = \sum_{i=1}^{n} \bar{F}_{Ti}; \quad \bar{M}_T = \sum_{i=1}^{n} \bar{M}_{Ti} \]  

(8)

Aerodynamic Model

The aerodynamic model presented in this study is applicable to a wide class of RLVs. The generic RLV configuration postulated has totally seven independently driven control surfaces; i.e. four elevons, a body flap, a rudder and a speed brake as shown in Fig.2.

The elevons are divided into four segments, right and left, inboard and outboard and are employed for pitch and roll control. For the control analysis and aerodynamic computations, the traditional control deflections are used as given below.

Elevator : \[ \delta_e = (\delta_{eL} + \delta_{eR})/2 \]  

(9)

Aileron : \[ \delta_a = (\delta_{eL} + \delta_{eR})/2 \]  

(10)

Where,

\[ \delta_{eL} = (\delta_{eL0} + \delta_{eLi})/2 \]  

(11)

\[ \delta_{eR} = (\delta_{eR0} + \delta_{eRi})/2 \]  

(12)

The aerodynamic characteristics are represented in body axis system. The aerodynamic forces are computed with the following equation.

\[ \bar{F}_A = \begin{bmatrix} -C_A \\ C_Y \\ -C_N \end{bmatrix} S_{ref} q_{dyn} \]  

(13)

Fig. 2 Elehon and body flap deflections for a generic RLV
The aerodynamic moments are computed with the equation given below.

\[ \mathbf{M} = \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix}_b \text{ref} S_{ref} q_{dyn} \]  

(14)

The aerodynamic force and moment coefficients are presented in a generalized form as addition of (1) basic coefficients as functions of \( M, \alpha, \beta \), (2) increments due to \( \delta_e, \delta_a, \delta_y, \delta_h \), landing gear deflections and ground effects and (3) dynamic derivatives due to body rates and rate of change of \( \alpha, \beta [3,5] \). The detailed nonlinear aerodynamic characteristics are as follows:

**Normal Force Coefficient**

\[ C_N = C_{N_0} (M, \alpha) + [\Delta C_{N_E} (M, \alpha, \delta_e) + \Delta C_{N_A} (M, \alpha, \delta_a)]_E \]

+ \( \Delta C_{N_F} (M, \alpha, \delta_e) + \Delta C_{N_A} (M, \alpha, \delta_a) + \Delta C_{N_FLX} \)

+ \( \Delta C_{N_{LG}} (\alpha, \theta) + \Delta C_{N_{GE}} (\delta_e \delta_a) \)  

(15)

\[ \Delta C_{N_{E\theta \delta_e \delta_a}} = K_{AE} \Delta C_{E_{\theta \delta_e \delta_a}} + (1 - K_{AE}) \Delta C_{E_{\theta \delta_e \delta_a}} \]  

(16)

\[ \Delta C_{N_{E\theta \delta_e \delta_a}} = \frac{1}{2} [K_{N_{\theta \delta_e \delta_a}} \Delta C_{N_{\theta \delta_e \delta_a}} + (1 - K_{N_{\theta \delta_e \delta_a}}) \Delta C_{N_{\theta \delta_e \delta_a}}] - \Delta C_{N_E} \]  

(17)

\[ \Delta C_{N_{FLX}} = \Delta C_{FLX} \cos \alpha + \Delta C_{FLX} \sin \alpha \]  

(18)

**Axial Force Coefficient**

\[ C_A = C_{A0} (M, \alpha) + [\Delta C_{AE} (M, \alpha, \delta_e) + \Delta C_{A_A} (M, \alpha, \delta_a)]_E \]

+ \( \Delta C_{A_F} (M, \alpha, \delta_e) + \Delta C_{A_A} (M, \alpha, \delta_a) + \Delta C_{A_{FLX}} (M, \alpha, \delta_e) \)  

+ \( \Delta C_{A_{LG}} (\alpha, \theta) + \Delta C_{A_{GE}} (\delta_e \delta_a) \)  

(19)

\[ \Delta C_{A_A} = \frac{1}{2} (\Delta C_{A_{E\theta \delta_e \delta_a}} + \Delta C_{A_{E\delta_e \delta_a \theta}} - \Delta C_{A_{E\theta \delta_a \delta_e}}) \]

(20)

\[ \Delta C_{A_{FLX}} = \Delta C_{D_{FLX}} \cos \alpha + \Delta C_{D_{FLX}} \sin \alpha \]  

(21)

**Side Force Coefficient**

\[ C_f = C_{f_0} (M, \alpha, \beta) + \Delta C_{f_0} (M, \alpha, \delta_e) + \Delta C_{f_0} (M, \alpha, \delta_a) \]

+ \( \Delta C_{f_F} (M, \alpha, \delta_e) + \Delta C_{f_A} (M, \alpha, \delta_a) + \Delta C_{f_{FLX}} (M, \alpha, \delta_e) \)

+ \( \Delta C_{f_{LG}} (\alpha, \theta) \)  

(22)

\[ \Delta C_{f_{LG}} = K_{f} \Delta C_{E_{\theta \delta_e \delta_a}} + (1 - K_{f}) \Delta C_{E_{\theta \delta_e \delta_a}} \]  

(23)

**Pitching Moment Coefficient about Centre of Mass**

\[ C_m = C_{m_0} (M, \alpha) + [\Delta C_{m_E} (M, \alpha, \delta_e) + \Delta C_{m_A} (M, \alpha, \delta_a)]_E \]

+ \( \Delta C_{m_F} (M, \alpha, \delta_e) + \Delta C_{m_A} (M, \alpha, \delta_a) + \Delta C_{m_{FLX}} (M, \alpha, \delta_e) \)

+ \( \Delta C_{m_{LG}} (\alpha, \theta) \)  

(24)

\[ \Delta C_{m_{LG}} = \Delta C_{m_{GE}} (\delta_e \delta_a) \]  

(25)

**Yawing Moment Coefficient about Centre of Mass**

\[ C_n = C_{n_0} (M, \alpha, \beta) + [\Delta C_{n_E} (M, \alpha, \delta_e) + \Delta C_{n_A} (M, \alpha, \delta_a)]_E \]

+ \( \Delta C_{n_F} (M, \alpha, \delta_e) + \Delta C_{n_A} (M, \alpha, \delta_a) + \Delta C_{n_{FLX}} (M, \alpha, \delta_e) \)

+ \( \Delta C_{n_{LG}} (\alpha, \theta) \)  

(26)
Rolling Moment Coefficient about Centre of Mass

\[ C_l = C_{l_0}(M, \alpha, \beta) + \Delta C_{l_b}(M, \alpha, \beta, \delta_b) + C_{l_p}(M, \alpha, \delta_p) \delta_p + \Delta C_{l_b}(M, \alpha, \beta, \delta_b) \delta_b + C_{l_p}(M, \alpha) \frac{p_{b_{ref}}}{2V_A} + C_{l_p}(M, \alpha) \frac{p_{b_{ref}}}{2V_A} + \Delta C_{l_{FLX}} \]

\[ + \Delta C_{l_{IG}}(\alpha, \beta) + \Delta C_{l_{GE}} \]

Subsystem Models

The RLV, in addition to the kinematics and dynamics, also requires correct models for the sensors and actuators, which interface with the navigation and control model of the complete system. The generic RLV has been assumed to contain both Flush Air Data System (FADS) and Space Integrated GPS/INS (SIGI) based sensors. The FADS measures pressures and computes total velocity, angle of attack and angle of sideslip [6]. The relation between the total pressure in the transducers and the measured pressure is modeled as a second order transfer function.

\[ p_t = \frac{\omega_n^2}{s^2 + 2\zeta_n \omega_n s + \omega_n^2} \]

where, \( p_t \) is the pressure sensed by the transducer and \( p_0 \) is real pressure on the surface of transducer. The natural frequency \( \omega_n \) and damping ratio \( \zeta \) are determined by the characteristic parameters of transducer and tubing, as well as the flight environmental parameters. SIGI is a navigation system that combines the outputs of an Inertial Navigation System (INS) and a Global Positioning System (GPS) receiver, to provide a full autonomous navigation solution for the RLV. The INS contains body-mounted accelerometers and rate gyro sensors for measuring the acceleration along body axes and angular velocity about body axes. The SIGI provides three navigation solutions namely: Inertial, GPS and combined. It performs the computation of the position of RLV and at the same time gives information about velocity, time, attitude and angular rate of the vehicle. To model the errors of SIGI sensor, the reentry trajectory is divided into three phases. The first phase starts from de-boost until black out, where the combined navigation solution is used. The second phase is black out phase, where the GPS is unavailable and only the inertial solution is used. The last phase is after black out, where again combined solution is used. The responses of accelerometers and rate gyro sensors and attitude computational loops are modeled using suitable second order transfer functions of the form of equation (28). Two error models are used corresponding to the two navigation solutions used during reentry [7]. In order to account for possible deficiencies in model, larger sensor errors are used during pure inertial navigation phase and smaller variations are used during combined phase. Errors are also introduced on the true navigational parameters viz., position, velocity and attitude in the following manner. In the inertial navigation phase, larger errors with non-zero bias errors are applied, while for the combined solution phase, much smaller variations on these parameters are used, along with zero bias errors. The control surfaces of generic RLV (i.e. aerodynamic control surfaces and RCS thrusters) are driven independently using electromechanical actuators. It is usually sufficient for the preliminary dynamic analysis to assume a second order transfer function for all the control surface responses, as given below:

\[ \frac{\delta}{\omega_n \sqrt{s^2 + 2\zeta \omega_n s + \omega_n^2}} \]

\( \omega_n \) and \( \zeta \) are natural frequency and damping ratio of the combined actuator-control surface system. For RCS, thruster characteristics e.g. on-delay, rise time, off-delay etc. are modeled.

Environmental Model

The Earth is assumed to be oblate for the computation of trajectory and flight parameters. The gravitational model for acceleration is given by the gravitational constant, along with the second, third and fourth harmonics, and expressed in ECI frame. Further, atmospheric model is constructed from look up tables, so as to become applicable to any general RLV mission. The atmospheric model includes the atmospheric properties viz., density, temperature, pressure, speed of sound and wind. These parameters are interpolated from the look up table as functions of altitude and used for the computation of various flight parameters.

Conclusions

The paper presents a general nonlinear formulation of reentry flight dynamics, using multiple frames of reference to minimize model complexities. Inertial reference frame for translational motion and quaternion representation for vehicle attitude are used to avoid computational singularities in the translational and angular
kinematic equations respectively. The atmosphere is modeled based on the Indian Standard Atmosphere of 2001. The Earth is modeled as an oblate spheroid and the gravitational potential is approximated with up to fourth Jeffery terms. The aerodynamic models are developed based on the compatible aerodynamic characteristics of such generic RLVs, available in literature. Simplified dynamic models of the navigation sensors, the FADS and SIGI, as well as actuators are approximated as second order dynamic models. The model presented here is considered to be general enough so that a wide variety of configurations and missions can be represented, using the given equations.

References


Definition of Relative Euler Angles

The vehicle orientation with respect to local reference frame is defined through three relative Euler angles viz., yaw angle \( \psi_R \), pitch angle \( \theta_R \) and roll angle \( \phi_R \).

Wind Frame for Aerodynamic Forces

The origin is center of mass of the vehicle, \( X_W \) is pointing forward, collinear with relative velocity vector, \( Z_W \) is pointing down, collinear with the lift force and \( Y_B \) completes the right-handed system.