NONLINEAR DYNAMIC ANALYSIS OF SKEW LAMINATES USING FINITE ELEMENT METHOD

M. K. Singha* , S. Prabhakar**, M. Ganapathi***

Abstract

Here, nonlinear dynamic behavior of composite skew plates is investigated using the finite element method. The formulation is based on shear deformation theory. The nonlinear governing equations of motion are solved using direct iteration procedure for free vibration case, and Newmark’s time integration technique is employed for dynamic response analysis. Few numerical examples are considered to highlight the influences of skew angle, fiber orientation, boundary condition and aspect ratio on the nonlinear dynamic characteristics of composite skew laminates.

Keywords: Composite skew plate, Free vibration, Transient response, FEM.

Introduction

Thin-walled structural components with relatively low flexural rigidity are used in the design of space vehicles. The study of nonlinear dynamic characteristics of such structural elements has attracted the attention of many researchers in recent years. These studies are reviewed and well documented by Liessa [1, 2], Bert [3], Sathyamoorthy [4], Chia [5], and Kapania and Raciti [6]. It is observed from the existing literature that the large amplitude flexural vibration of rectangular/circular plates has been dealt in details, whereas limited work has been focused on non-rectangular cases.

The plates with non-rectangular plan-forms like skew plates find wide engineering applications such as skewed bridge decks, aircraft wings [7], vehicle bodies and ship hulls. Though a considerable amount of literature is available on the linear free vibration of isotropic skew plates, such analysis pertaining to the laminated composite skew plates has received relatively little attention. The notable recent contributions pertaining to linear vibration behavior of laminated composite skew plates are dealt with Refs. 8-16. Iyengar and Umaretiya [8] have studied a hybrid laminated skew plate using Galerkin’s method. Malhotra et al. [9] have carried out the vibration of rhombic orthotropic plates using parallellogrammic orthotropic plate finite element whereas Krishnan and Deshpande [10] have examined the free vibration of thin cantilever skew laminates employing finite element method, based on discrete Kirchhoff theory. Kapania and Singhvi [11], and Anlas and Goker [12] have analytically solved the problem using Ritz method. Wang [13] has studied using B- spline Rayleigh-Ritz method, while Han and Dickinson [14] have applied a Ritz Hierarchical finite element in analyzing the laminated skew plates. A numerical approach with assumed solution considering Green function is introduced in the work of Hosokawa et.al. [15]. Reddy and Palaninathan [16] have dealt with the problem using high precision plate bending finite element. Makhecha et.al. [17] and Wang et.al. [18] have studied the vibration characteristics of skew sandwich plates with laminated facings. It may be opined that most of these work are carried out based on classical plate theory and the analysis devoted to the nonlinear dynamic behavior of skew plates is rather meager in the literature.

Nowinski [19] and most recently Ray et al. [20] have analytically studied the nonlinear vibration behavior of isotropic skew plates. Sathyamoorthy and Pandalai [21] have analyzed single layer orthotropic skew plates for moveable in-plane edges and Berger approximation for immovable in-plane edge conditions. Prathap and Varadan [22], and Sathyamoorthy and Chia [23] have investigated large amplitude vibration of anisotropc skew plates using the Galerkin method on the basis of a single term assumed vibration mode. All these investigations introduce assumed mode while solving the problem. It is known that one-term approximation for the free vibration
mode is insufficient for rectangular as well as skew plates, and such results lose accuracy with increase in aspect ratio and skew angles. Also, to the best of the authors' knowledge, the work on the nonlinear dynamics of laminated anisotropic composite skew plates is not commonly yet available in the literature.

For accurate solution, the assumed mode shape in the analytical approach should have more terms that lead to unwieldy algebraic and more numerical work. Numerical methods such as finite element procedure is preferable over the analytical methods, while studying the large amplitude dynamic analysis of structures, as there is no need for an a priori assumption of the mode shapes, and the solution itself predict the mode shapes [24]. Such analysis for skew plates appears to be lacking in the literature.

In the present paper, a four-noded shear flexible quadrilateral high precision plate bending element developed recently [25, 26] is extended to analyze the nonlinear dynamic behavior of laminated composite skew plates. The formulation includes in-plane and rotary inertia effects. The nonlinear governing equations of motion are solved by direct iteration procedure for free vibration case whereas Newmark’s time integration technique is employed for dynamic response analysis. The formulation developed here is validated with the available analytical/numerical results. Limited numerical work has been carried out to study the influences of the skew angle, lay-up, aspect ratio, and boundary conditions on the nonlinear free and forced vibration behavior of laminated skew plates.

Formulation

The displacement components at an arbitrary point \((x, y, z)\) of a quadrilateral plate can be expressed as

\[
\begin{align*}
\nu (x, y, z) &= \nu_0 (x, y) - z \left( w_x + \gamma_y (x, y) \right) \\
w (x, y, z) &= w_0 (x, y)
\end{align*}
\]

From the theory of large displacement, the membrane strains, curvatures, and shear strains can be written as

\[
\begin{align*}
\varepsilon_x &= u_{,x} + \frac{1}{2} w_{,xx} \\
\varepsilon_y &= u_{,y} + \frac{1}{2} w_{,yy} \\
\gamma_{xy} &= u_{,x} + u_{,y} + \frac{1}{2} w_{,xy}
\end{align*}
\]

\[
\begin{align*}
k_x &= w_{,xx} + \gamma_{,x} \\
k_y &= w_{,yy} + \gamma_{,y} \\
k_{xy} &= 2w_{,xy} + \gamma_{,x} + \gamma_{,y}
\end{align*}
\]

\[
\begin{align*}
\gamma_{xx} &= -\gamma_x \\
\gamma_{yy} &= -\gamma_y
\end{align*}
\]  (2)

In the present work, oblique coordinate system (Fig. 1) is used. The relationships between global coordinate system \((x_G, y_G, z_G)\) and oblique coordinate system \((x, y, z)\) are as follows

\[
\begin{align*}
x_G &= x + y \sin \psi \\
y_G &= y \cos \psi \\
z_G &= z
\end{align*}
\]  (3)

The first derivatives of a function \(f(x, y)\) may be expressed in global Cartesian coordinate as

\[
\begin{bmatrix}
\frac{\partial f}{\partial x_G} \\
\frac{\partial f}{\partial y_G}
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 \\
-\tan \psi & \sec \psi
\end{bmatrix}
\begin{bmatrix}
\frac{\partial f}{\partial x} \\
\frac{\partial f}{\partial y}
\end{bmatrix}
\]  (4)

Fig. 1 Oblique co-ordinate system for the skew plate
Similarly, the second derivatives of any function \( f(x, y) \) may be expressed as

\[
\begin{bmatrix}
\frac{\partial^2 f}{\partial x^2} \\
\frac{\partial^2 f}{\partial x \partial y} \\
\frac{\partial^2 f}{\partial y^2}
\end{bmatrix}
= \begin{bmatrix}
\tan^2 \psi & \sec \psi - \sec \psi \tan \psi & 2 \tan \psi \\
\tan^2 \psi & \sec \psi \tan \psi & 0 \\
-2 \tan \psi & \sec \psi & 2 \sec \psi
\end{bmatrix}
\begin{bmatrix}
\frac{\partial^2 f}{\partial x^2} \\
\frac{\partial^2 f}{\partial x \partial y} \\
\frac{\partial^2 f}{\partial y^2}
\end{bmatrix}
\]

The internal strain energy \( U \), and the work done \( W \) by the external load \( q(x, y) \) acting over the surface of the plate can be written as

\[
U(\delta) = \frac{1}{2} \int_A \left[ \varepsilon \begin{bmatrix} T_A \ v \ B \ k \end{bmatrix} \varepsilon \right] + \left[ \varepsilon \begin{bmatrix} T_B \ k \end{bmatrix} \varepsilon \right] + \left[ \varepsilon \begin{bmatrix} T_S \ k \end{bmatrix} \varepsilon \right] \ dA
\]

\[
W(\delta) = \int_A q(x, y) \ dA
\]

where \([A], [B], [D], \) and \([S] \) are extensional, bending-extensional, bending, and shear stiffness coefficients respectively. For a composite laminate of thickness \( h \), comprising of \( N \) layers with stacking angles \( \theta_i \) \( (i = 1, 2, \ldots, N) \) and layer thicknesses \( h_i \) \( (i = 1, 2, \ldots, N) \), the necessary expression to compute the stiffness coefficients may be expressed as [27]

\[
(A_{ij}, B_{ij}, D_{ij}) = \sum_{i=1}^{N} \int_{h_{k-1}}^{h_k} [Q_{ij}]^k (1, z, z^2) \ dA
\]

\[
S_{ij} = \sum_{i=1}^{N} \int_{h_{k-1}}^{h_k} [Q_{ij}] \ dA
\]

represents plane stress reduced stiffness matrix of the \( k^{th} \) laminate of the plate. The stacking angles \( \theta_i \) \( (i = 1, 2, \ldots, N) \) are measured with respect to \( x \)-axis.

The kinetic energy of the plate is given by

\[
T(\delta) = \frac{1}{2} \int \left[ (u_0^2 + v_0^2 + \omega^2) \rho + (\Theta_x^2 + \Theta_y^2) \rho z^2 \right] \ dV
\]

Substituting equations (6) and (8) in Lagrange's equation of motion, one obtains the governing equation as

\[
\left[ (K_n) + [M] \right] \ddot{\delta} = \{ F \}
\]

where \([M] \) is mass matrix, \([K_n] \) and \([M] \) are the linear and nonlinear stiffness matrices and \{ \} is the vector of nodal displacements and \{F\} is the vector of nodal forces. The governing equation (9) is solved using finite element approach based on \( C^0 \) continuous element developed recently [25, 26].

Here, a four-noded quadrilateral plate element with ten degrees of freedom per node, namely \( u, v, w, w_x, W_y, W_{xx}, W_{xy}, W_{yy}, \gamma_x \) and \( \gamma_y \) are used. The linear polynomial shape functions are employed to describe the field variables corresponding to in-plane displacements \( (u_0, v_0) \) and rotations due to shear of the middle surfaces \( (\gamma_x, \gamma_y) \), whereas quartic polynomial function is considered for the lateral displacement \( \omega \) and are expressed as follows.

\[
u_0 = [1, x, y, xy] \ [c_i],
\]

\[
u_0 = [1, x, y, xy] \ [c_i],
\]

\[
u_0 = [1, x, y, xy] \ [c_i],
\]

\[
u_0 = [1, x, y, xy] \ [c_i],
\]

\[
u_0 = [1, x, y, xy] \ [c_i],
\]

\[
u_0 = [1, x, y, xy] \ [c_i],
\]

The full integration scheme with 6 x 6 Gaussian integration rule is adopted for computing the element mass matrix, and stiffness matrices. This element is free from locking syndrome. It has good convergence properties and has no spurious rigid modes [25, 26].

Substituting characteristics of the time function at the point of reversal of motion (maximum displacement)

\[
\dot{\delta} = -\omega^2 \delta
\]
in equation (9), and assuming \( \{F\} = \{0\} \) will lead to the following nonlinear algebraic equation for the free vibration problem,

\[ \left( K_L + K_{nc} \right) \{ \delta \} - \omega^2 \{ M \} \{ \delta \} = \{ 0 \} \]  

where, \( \omega \) is the natural frequency. The frequency-amplitude relation is obtained by solving the equation (12) iteratively as highlighted in Ref. [28].

For the forced vibration analysis, Newmark’s time integration technique is employed to solve Equation (9). A convergence study is carried out to select a time step, which yields a stable and accurate solution. The critical time step as given in Ref. 29 is

\[ \Delta t = \left[ \frac{\rho (1 - \mu^2)/E_t}{2 + (1 - \mu^2) \frac{h^2}{12} \left( 1 + 1.5 \frac{\Delta x^2}{h^2} \right)} \right]^{1/2} \Delta x \]  

(13)

where \( \Delta x \) is the minimum distance between the element node points and \( E_t \) is Young’s modulus in the direction of fiber.

Results and Discussions

The present study is focused on the nonlinear free and forced vibration behaviors of laminated composite skew plates. The material properties, unless specified otherwise, used in the present analysis are

\[ E_t/E_r = 40.0, \quad G_{tt}/E_r = 0.6, \quad G_{tr}/E_r = 0.5, \quad \nu_{tr} = 0.25 \]

\[ E_r = 1 \times 10^6 \text{ N/cm}^2, \quad \rho = 8 \times 10^{-6} \text{ N s}^2/\text{cm}^4. \]

where \( E, G \) and \( \nu \) are Young’s modulus, shear modulus and Poisson’s ratio respectively. Subscripts L and T represent the longitudinal and transverse directions respectively with respect to fibers. All the layers are of equal thickness. The boundary conditions considered here are:

Simply supported case (SS):

\[ u_0 = v_0 = w = 0 \quad \text{at} \quad x = 0, \quad a \text{ and } y = 0, b \]

Clamped edge (CC):

\[ u_0 = v_0 = w = 0, \quad w_x = 0 \quad \text{at} \quad x = 0, a \]

\[ u_0 = v_0 = w = 0, \quad w_y = 0 \quad \text{at} \quad y = 0, b \]

Before proceeding for the detailed study on nonlinear dynamic behavior of laminated composite skew plates, the formulation developed here, is validated against its free flexural vibration. The non-dimensional natural frequencies \( (\bar{\omega} = \omega a^2/\pi^2 h \sqrt{\rho/Er} ; a \text{ and } h \text{ are length and thickness of the plate}) \) obtained for simply supported, and clamped 5-layered cross-ply \([90^\circ/0^\circ/90^\circ/0^\circ/90^\circ]\) skew laminates are presented in Table-1 and 2, along with the analytical solutions of Wang [13] and they match very well. It is also observed from Table-1 that the element employed here has a good convergence property and thus, a 10 x 10 mesh is found to be adequate to model the full plate. It is also evident from these Tables that the present element does not exhibit any locking syndrome for ex-

<table>
<thead>
<tr>
<th>Skew Angle</th>
<th>Mesh Size</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
<th>Mode 6</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>Present Study</td>
<td>4 x 4</td>
<td>1.9074</td>
<td>3.9507</td>
<td>6.5025</td>
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<td>8.0497</td>
</tr>
<tr>
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<td>3.9721</td>
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<td>7.6468</td>
<td>8.1462</td>
</tr>
<tr>
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<td>10.9638</td>
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<td>8.4717</td>
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<td>12.1070</td>
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extremely thin plate case \((a/h=1000)\). The variation of non-linear frequency ratio \((\omega_{NL}/\omega_L)\) with respect to the non-dimensional maximum amplitude \((w/h)\) is evaluated and shown in Table-2 and are found to be in excellent agreement with the existing solutions [28, 30]. It is observed from Table-3, that, there is a drop in the frequency value at certain amplitude of vibration and then again it increases with amplitude exhibiting hardening behavior. This is due to a redistribution of the mode shape at higher amplitude of vibration, thus leading to a change in stiffness value and been explained later. In the next section, the study of nonlinear behavior of composite skew plates, for which the results are not available in the literature, have been presented.

### Table 2: Comparison of non-dimensional linear natural frequency \((\bar{\omega} = \omega a^2/\pi^2 h \sqrt{\rho/E_T})\) of 5-layered \([90^\circ/0^\circ/90^\circ/0^\circ/90^\circ]\) clamped skew laminates for various skew angle \((a/b=1; a/h = 1000)\)

<table>
<thead>
<tr>
<th>Skew angle</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>4.2371</td>
<td>6.6893</td>
<td>10.4411</td>
<td>11.4309</td>
<td>11.7699</td>
<td>15.1179</td>
</tr>
</tbody>
</table>

### Table 3: Comparison of non-linear frequency ratio \((\omega_{NL}/\omega_L)\) of simply supported square plate \((E_L/E_T = 3.0, G_{LT}/E_T = 0.5, \nu_{LT} = 0.25; a/b = 1; a/h = 1000)\)

<table>
<thead>
<tr>
<th>w/h</th>
<th>Isotropic Plate</th>
<th>Orthotropic Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.02538</td>
<td>1.02599</td>
</tr>
<tr>
<td>0.4</td>
<td>1.09830</td>
<td>1.10027</td>
</tr>
<tr>
<td>0.6</td>
<td>1.21098</td>
<td>1.21402</td>
</tr>
<tr>
<td>0.8</td>
<td>1.35448</td>
<td>1.35735</td>
</tr>
<tr>
<td>1.0</td>
<td>1.52101</td>
<td>1.52192</td>
</tr>
<tr>
<td>1.2</td>
<td>1.70457</td>
<td>-</td>
</tr>
<tr>
<td>1.4</td>
<td>1.90072</td>
<td>-</td>
</tr>
<tr>
<td>1.6</td>
<td>1.63426</td>
<td>-</td>
</tr>
<tr>
<td>1.8</td>
<td>1.64660</td>
<td>-</td>
</tr>
<tr>
<td>2.0</td>
<td>1.66476</td>
<td>-</td>
</tr>
</tbody>
</table>
seen from Fig. 2 that the vibration mode loses its symmetry and the location of maximum displacement shifts towards one side of the plate. This trend is observed for all the values of skew angle. However, for clamped boundary condition, the amplitude corresponding to the mode jump/mode redistribution is little higher. Furthermore, for the chosen amplitude, it can be noted that, with the increase in skew angle, the degree of nonlinearity is high before the occurrence of mode redistribution particularly for the case of clamped plate. Similar phenomenon is reported earlier by the several authors [25, 26, 31], while

![Fig. 2 The modal shapes before and after mode jump/mode redistribution of a simply supported square 5-layered $[0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]$ composite plate ($a/b = 1; a/h = 100$)](image)

<table>
<thead>
<tr>
<th>$w/h$</th>
<th>$0^\circ$</th>
<th>$15^\circ$</th>
<th>$30^\circ$</th>
<th>$45^\circ$</th>
<th>$60^\circ$</th>
</tr>
</thead>
<tbody>
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<td>0.0</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
</tr>
<tr>
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<td>1.03911</td>
<td>1.03753</td>
<td>1.03763</td>
<td>1.04216</td>
</tr>
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<td>1.60768</td>
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<tr>
<td>1.0</td>
<td><strong>1.48306</strong></td>
<td>1.75032</td>
<td>1.72179</td>
<td>1.71460</td>
<td>1.38708</td>
</tr>
<tr>
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<td>1.55815</td>
<td>1.49200</td>
<td>1.43778</td>
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</tbody>
</table>

Table-4: The non-linear frequency ratios ($\omega_{NL}/\omega_L$) of simply supported cross-ply $[0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]$ skew plate ($a/b=1; a/h = 100$)
Table-5: The non-linear frequency ratios ($\omega_{NL}/\omega_{OL}$) of a clamped cross-ply [0°/90°/0°/90°] skew plate (a/h=1; a/h = 100)

<table>
<thead>
<tr>
<th>W/h</th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
</tr>
<tr>
<td>0.0</td>
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<td>1.01066</td>
<td>1.01119</td>
<td>1.01237</td>
<td>1.01422</td>
</tr>
<tr>
<td>0.2</td>
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<td>1.04404</td>
<td>1.04859</td>
<td>1.05577</td>
</tr>
<tr>
<td>0.6</td>
<td>1.09101</td>
<td>1.09218</td>
<td>1.09661</td>
<td>1.10635</td>
<td>1.12175</td>
</tr>
<tr>
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<td>1.15890</td>
<td>1.16629</td>
<td>1.18254</td>
<td>1.20833</td>
</tr>
<tr>
<td>0.9</td>
<td>1.23682</td>
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<td>1.25034</td>
<td>1.27391</td>
<td>1.31163</td>
</tr>
<tr>
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<td>1.33194</td>
<td>1.34616</td>
<td>1.37753</td>
<td>1.42809</td>
</tr>
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<td>1.31566</td>
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<tr>
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<td>1.30605</td>
<td>1.31013</td>
<td>1.31875</td>
<td>1.32810</td>
</tr>
</tbody>
</table>

Table-6: The non-linear frequency ratios ($\omega_{NL}/\omega_{OL}$) of simply supported cross-ply [0°/90°/0°/90°] skew plate for various aspect ratios (a/h = 100)

<table>
<thead>
<tr>
<th>W/h</th>
<th>Skew angle = 0°</th>
<th>Skew angle = 30°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b/a=1.0</td>
<td>b/a=1.5</td>
</tr>
<tr>
<td>0.0</td>
<td>1.00000</td>
<td>1.00000</td>
</tr>
<tr>
<td>0.2</td>
<td>1.03998</td>
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<td>1.15243</td>
<td>1.13194</td>
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<tr>
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<td>1.23073</td>
<td>1.20179</td>
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<tr>
<td>0.6</td>
<td>1.32081</td>
<td>1.28366</td>
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<tr>
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<td>1.48306</td>
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<td>1.67886</td>
<td>1.37907</td>
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investigating the thermal postbuckling of composite plates and it is termed as secondary instability.

The variation of nonlinear frequency ratio ($\omega_{NL}/\omega_L$) with the non-dimensional amplitude of vibration ($w/h$) for a 5-layered angle-ply [0°/90°/0°/90°/0°] simply supported skew plate has been reported in Table-6 for various aspect ratios. The mode jump/redistribution phenomenon is observed here also, but their occurrence with respect to amplitude of vibration is different for different skew angle. In general, it occurs at lower amplitude with increase in aspect ratio.

Figures 3 (a) and (b) show the variation of nonlinear frequency ratio ($\omega_{NL}/\omega_L$) for a simply supported 2-layered cross-ply [0°/90°] and angle-ply [45°/-45°] skew plate ($a/h=1$) respectively. The analysis is carried out considering different levels of amplitude of vibration in the inward ($w/h$ is negative) and outward ($w/h$ is positive) motions of plate. For the case of square plate (skew angle = 0°), both angle-ply and cross-ply laminates show hardening behavior and the mode redistribution phenomenon (drop in the frequency value) is also occurred, irrespective of the direction of motion of plate. It is further more seen that the nonlinear behavior is symmetric with respect to the plate motion. However, for the case of skew plate, the unsymmetric laminates show different level of hardening behavior due to the change in bending-stretching stiffness coupling and it depends on direction of amplitudes ($w/h$). It is also noticed that, for the inward motion case, softening type of nonlinearity is initially predicted and then, with increase in amplitude, the nonlinearity changes into hardening trend.

Next, the nonlinear transient response analyses of 5-layered cross-ply [0°/90°/0°/90°/0°] and angle-ply [45°/-45°/45°/-45°/45°] skew plates are considered. The spatial distribution of the transverse step load, considered here are

$$q(x, y) = q_0 \sin \left(\frac{\pi x}{a}\right) \sin \left(\frac{\pi y}{b}\right)$$

All the initial conditions are assumed to be zero. Newmark’s time integration technique is adopted here. A convergence study is carried out to select a time step, which yields a stable and accurate solution. The transverse displacement $w$ at the center (a/2, b/2) of the plate are plotted with the nonlinear response curves. It is observed that the nonlinear response is significantly different from the linear response, both in amplitude and period. The degree of nonlinearity increases with the increase of load

Conclusions

Nonlinear dynamic analysis of composite skew plate has been studied using a four-noded shear flexible quadrilateral high precision plate bending element. A detailed study is made to investigate the effect of skew angle, fiber orientation, aspect ratio, and boundary conditions on the nonlinear frequency ratio and the dynamic characteristics.
of composite skew plate. From the present numerical work, the following few conclusions may be drawn:

- The nonlinear frequency increases with the amplitude of vibration up to a certain value of amplitude. Further increase of amplitude leads to redistribution of mode shapes due to change in stiffness values.
- Mode jump / mode redistribution phenomenon occurs for all the cases irrespective of skew angle and ply-orientation. However, its occurrence depends on the skew angle, fiber orientation and boundary conditions.
- The symmetric fundamental mode shape becomes unsymmetric at the point of "mode jump/mode redistribution" and the maximum displacement shifts from center of laminates.
- The degree of nonlinearity is more for the cross-ply plate than those of isotropic or orthotropic cases.
- The degree of nonlinearity, in general, increases with skew angles and its rate of change depends on the occurrence of mode redistribution phenomenon.
- For the case of un symmetric skew laminates, in general, the hardening behavior depends on the direction of amplitude.
- The nonlinearity has significant effects on amplitude and period of the response compared to linear case.

References
Fig. 5 Variation of transfer displacement $w(x/2, b/2)$ of a 5-layered angle-ply plate [45°/−45°/45°/−45°/45°] subjected to a step load $q(x, y) = q_0 \sin (\pi x/a) \sin (\pi y/b)$; $(a/b = 1; a/h = 100)$


