INFLUENCES OF FUNCTIONALLY GRADED MATERIALS ON SUPersonic PANEL FLUTTER

T. Prakash*, M. Ganapathi**, Maloy K. Singha**

Abstract

Here, the supersonic flutter behavior of flat panels made of functionally graded materials is studied. The structural model is based on first-order shear theory and material properties are assumed to be temperature dependent and graded in the thickness direction. The aerodynamic force is evaluated by considering the first-order high Mach number approximation to linear potential flow theory. The variation of critical aerodynamic pressure is highlighted considering different parameters such as plate thickness, aspect ratio, power law index of functionally graded materials, temperature-dependent material properties. It is seen that the volume fraction index plays a significant role on the critical flutter speed under thermal environment.

Keywords: Functionally graded plate, Supersonic flutter, Finite element, Aspect ratio, Temperature, Volume fraction index.

Introduction

Functionally graded materials have been receiving more attention in the recent years, especially in the aerospace industry because of their ability to withstand high temperature gradient. These materials are microscopically inhomogeneous composites usually made from a mixture of metals and ceramics. The ceramic material provides high temperature resistance due to its low thermal conductivity, while the ductile metal component prevents fracture due to thermal stresses. By gradually varying the volume fraction of constituent materials, their material properties exhibit a smooth and continuous change from one surface to another surface of the plate [1,2]. The increased effort towards integrating these materials in the construction of aerospace structures has necessitated investigating the dynamic response of functionally graded structures.

Most of the work available on functionally graded plates in the literature have been restricted to static analysis. Limited work has been reported on dynamic analysis of functionally graded plates and some of them are briefly mention here [3-7]. Cheng and Batra [3] have studied the steady state vibration of a simply supported functionally graded polygonal plate with temperature independent material properties. Recently, Yang and Shen [4] have examined the vibration characteristic and transient response of functionally graded plates in thermal environments. He et al. [5] have presented the vibration control of FGM thin plate with integrated piezoelectric sensors and actuators. An attempt is also made in the literature to bring out the nonlinear dynamic behavior of plates made of functionally graded materials [6,7]. Praveen and Reddy [6] have analyzed the dynamic response of functionally graded ceramic metal plates through the finite element method whereas Yang, et al. [7] investigated the large amplitude vibration of thermo-electro-mechanically stressed FGM laminated plates. However, the investigation pertaining to dynamic instability of functionally graded structures is rather sparsely treated in the literature compared to vibration analysis. Ref. [8] concerns with the parametric resonance of functionally graded plates subjected to periodic axial load.

* No. 11B, Sirudhunai Nayanar St, Palayamkottai, Tirunelveli Dist, Tamilnadu-627 002, India
+ No. 8, Second Cross, Kumarar Nagar, Puttur (Post), Tiruchirapalli, Tamilnadu-620 015, India
Email: mganapathi@rediffmail.com
** Janapara, Nazargunj, Midnapore, West Bengal-721 101, India.
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The study of such materials for the thermo-aerodynamic behavior is important in the design of flight vehicles. The flutter phenomenon is one kind of dynamic instability encountered during the flight of aerospace vehicles. It is the self-excited oscillation of the external skin of a vehicle when exposed to airflow along its surface and there is a critical pressure above that the panel motion becomes unstable and grows exponentially with time. This study pertaining to isotropic and laminated composite cases have received considerable attention in the literature and are reviewed by Dowell [9] and recently by Bismark-Nasr [10,11]. However, to the best of authors' knowledge, the investigation of the flutter behavior of functionally graded flat panels has not been yet accomplished in the literature. As the use of panels made of functionally graded materials is expected to increase in the coming years in the aerospace industry, it is worth investigating the dynamic stability characteristics of such structures exposed to aerodynamic flow.

Here, an eight-noded shear flexible quadrilateral plate-bending element developed recently [12, 13] is extended to analyze the supersonic panel flutter behavior of functionally graded plates. The aerodynamic force is evaluated considering the first-order high Mach number approximation to linear potential flow theory. The QR algorithm is employed for the solution of complex eigenvalue problem. The formulation developed herein is validated with the available solutions. A detailed study has been made to highlight the influences of the plate thickness, aspect ratio and power law index on the flutter behavior of FGM plates.

**Theoretical Development and Formulation**

A functionally graded rectangular plate (length \(a\), width \(b\), and thickness \(h\)) made of a mixture of ceramics and metals is considered with the coordinates \(x, y\) along the in-pane directions and \(z\) along the thickness direction. The material in top surface \((z = h/2)\) of the plate and in bottom surface \((z = -h/2)\) of the plate is ceramic and metal, respectively. The effective material properties \(P\), such as Young's modulus \(E\), and thermal expansion coefficient \(\alpha\), can be written as [6]

\[
P = P_c V_c + P_m V_m
\]

where \(P_c\) and \(P_m\) are the material properties of the ceramic rich top surface and metal rich bottom surface, respectively. \(V_c\) and \(V_m\) are volume-fractions of ceramic and metal respectively and are related by

\[
V_c + V_m = 1
\]

The properties of the plate are assumed to vary through the thickness. The property variation is assumed to be in terms of a simple power law. The volume fraction \(V_c\) is expressed as

\[
V_c(z) = \left(\frac{2z + h}{2h}\right)^k
\]

where \(k\) is the volume fraction exponent \((k \geq 0)\). The material properties \(P\) that are temperature dependent can be written as

\[
P = P_0 (P_c T^{-1} + P_1 T + P_2 T^2 + P_3 T^3)
\]

where \(P_0, P_1, P_2, P_3\) are the coefficients of temperature \(T(K)\) and are unique to each constituent.

From Eqs. (1) - (4), the modulus of elasticity \(E\), the coefficient of thermal expansion \(\alpha\), the density \(\rho\) and the thermal conductivity \(K\) are written as

\[
E(z,T) = (E_c(T) - E_m(T)) \left(\frac{2z + h}{2h}\right)^k + E_m(T)
\]

\[
\alpha(z,T) = (\alpha_c(T) - \alpha_m(T)) \left(\frac{2z + h}{2h}\right)^k + \alpha_m(T)
\]

\[
\rho(z) = (\rho_c - \rho_m) \left(\frac{2z + h}{2h}\right)^k + \rho_m
\]

\[
K(z) = (K_c - K_m) \left(\frac{2z + h}{2h}\right)^k + K_m
\]

Here the mass density \(\rho\) and thermal conductivity \(K\) are assumed to be independent of temperature. The Poisson's ratio \(\nu\) is assumed to be a constant \(\nu(z) = \nu_0\).

The temperature variation is assumed to occur in the thickness direction only and the temperature field is considered constant in the \(xy\) plane. In this case, the
temperature through thickness is governed by the one-dimensional Fourier equation of heat conduction:

$$\frac{d}{dz} \left[ K(z) \frac{dT}{dz} \right] = 0, \quad T = T_c \text{ at } z = h/2$$

$$T = T_m \text{ at } z = -h/2$$

The solution of Eq. (6) is obtained by means of polynomial series [14] and given by

$$T(z) = T_m + (T_c - T_m) \eta(z)$$

where \( \eta(z) = \frac{1}{C} \left[ \frac{(2z+h)}{2h} - \frac{K_m}{(k+1)K_m} \left( \frac{(2z+h)}{2h} \right)^{k+1} \right]$$

$$+ \frac{K_m^2}{2(k+1)K_m^2} \left( \frac{(2z+h)}{2h} \right)^{2k+1} - \frac{K_m^3}{3(k+1)K_m^3} \left( \frac{(2z+h)}{2h} \right)^{3k+1}$$

$$+ \frac{K_m^4}{4(k+1)K_m^4} \left( \frac{(2z+h)}{2h} \right)^{4k+1} - \frac{K_m^5}{5(k+1)K_m^5} \left( \frac{(2z+h)}{2h} \right)^{5k+1}$$

and \( K_m = K_c - K_m \).

Using Mindlin formulation, the displacements \( u, v, w \) at a point \( (x, y, z) \) in the plate (Fig. 1) from the medium surface are expressed as functions of mid-plane displacements \( u_0, v_0 \) and \( w \), and independent rotations \( \theta_x \) and \( \theta_y \) of the normal in \( xz \) and \( yz \) planes, respectively, as

$$u(x, y, z, t) = u_0(x, y, t) + z\theta_x(x, y, t)$$

$$v(x, y, z, t) = v_0(x, y, t) + z\theta_y(x, y, t)$$

$$w(x, y, z, t) = w_0(x, y, t)$$

where \( t \) is the time. The strains in terms of mid-plane deformation can be written as

$$\{\varepsilon\} = \begin{bmatrix} \varepsilon_p \\ 0 \\ \varepsilon_s \end{bmatrix} \begin{bmatrix} z \varepsilon_b \end{bmatrix}$$

The mid-plane strains \( \{\varepsilon_p\} \), bending strains \( \{\varepsilon_b\} \) and shear strains \( \{\varepsilon_s\} \) can be obtained using Eq. (8).

The membrane stress resultants \( \{N\} \) and the bending stress resultants \( \{M\} \) can be related to the membrane strains \( \{\varepsilon_p\} \) and bending strains \( \{\varepsilon_b\} \) through the constitutive relations by

$$\{N\} = [A_1] \{\varepsilon_p\} + [B_1] \{\varepsilon_b\} - \{N^T\}$$

$$\{M\} = [B_2] \{\varepsilon_p\} + [D_2] \{\varepsilon_b\} - \{M^T\}$$

where the matrices \([A_1], [B_1], \) and \([D_2]\) \((i, j = 1, 2, 6)\) are the extensional, bending-extensional coupling and bending stiffness coefficients and are defined as \([A_1], [B_1], [D_2]\) = \( \int_{-h/2}^{h/2} \{Q_j\}(1, z, z^2)dz \).

The thermal stress resultant \( \{N^T\} \) and moment resultant \( \{M^T\} \) are
\[ N^T = \int_{-h/2}^{+h/2} \begin{bmatrix} \alpha(z,T) \\ 0 \end{bmatrix} \alpha(z,T) \Delta T(z) \, dz \] (12)
\[ M^T = \int_{-h/2}^{+h/2} \begin{bmatrix} \alpha(z,T) \\ 0 \end{bmatrix} \alpha(z,T) \Delta T(z) \, dz \] (13)

where the thermal coefficient of expansion \( \alpha(z,T) \) is given by Eq. (5), and \( \Delta T(z) = T(z) - T_0 \) is temperature rise from the reference temperature \( T_0 \) at which there are no thermal strains.

Similarly the transverse shear force \( \{Q\} \) representing the quantities \( Q_{xz}, Q_{xy} \) is related to the transverse shear strains \( \{\varepsilon_y\} \) through the constitutive relations as
\[
\{Q\} = [E_{ij}] \{\varepsilon_y\} \tag{14}
\]
where
\[
E_{ij} = \int_{-h/2}^{+h/2} [\overline{Q}_{ij}] \kappa_i \kappa_j \, dz
\]

Here \( [E_{ij}] \) \((i, j = 4, 5)\) are the transverse shear stiffness coefficients, \( \kappa_i \) is the transverse shear coefficient for non-uniform shear strain distribution through the plate thickness. \( \overline{Q}_{ij} \) are the stiffness coefficients and are defined as
\[
\overline{Q}_{11} = \overline{Q}_{22} = \frac{E(z,T)}{1 - \nu^2}; \quad \overline{Q}_{12} = \nu \frac{E(z,T)}{1 - \nu^2};
\]
\[
\overline{Q}_{16} = \overline{Q}_{26} = 0; \quad \overline{Q}_{44} = \overline{Q}_{55} = \overline{Q}_{66} = \frac{E(z,T)}{2(1 + \nu)} \tag{15}
\]

where the modulus of elasticity \( E(z,T) \) is given by Eq. (5).

The strain energy functional \( U \) is given as
\[
U(\delta) = (1/2) \int_A \left\{ [\varepsilon_y] \begin{bmatrix} A_{ij} & 0 \\ 0 & A_{ij} \end{bmatrix} [\varepsilon_y] + [\varepsilon_y] \begin{bmatrix} B_{ij} & 0 \\ 0 & B_{ij} \end{bmatrix} [\varepsilon_y] + [\varepsilon_y] \begin{bmatrix} D_{ij} & 0 \\ 0 & D_{ij} \end{bmatrix} [\varepsilon_y] \right\} \, dA
\]
\[
\left( \{\varepsilon_y\} \right) \{\varepsilon_y\} \} = \{N^T\} \{\varepsilon_y\} - \{N^T\} \{\varepsilon_y\} \{M^T\} \] (16)

where \( \delta \) is the vector of the degree of freedom associated to the displacement filed in a finite element discretisation.

The kinetic energy of the plate is given by
\[
T(\delta) = (1/2) \int_A \left[ p(\dot{u}_x^2 + \dot{u}_y^2 + \dot{w}_z^2) + I \left( \dot{\theta}_x^2 + \dot{\theta}_y^2 \right) \right] \, dA 
\tag{17}
\]

where \( p = \int \rho(z) \, dz \), \( I = \int z^2 \rho(z) \, dz \) and \( \rho(z) \) is mass density which varies through the thickness of the plate and is given by Eq. (5).

The panel is subjected to temperature field and this, in turn, results in-plane stress resultants \( (N_{xx}^h, N_{yy}^h, N_{xy}^h) \). Thus, the potential energy due to pre-buckling stresses \( (N_{xx}^h, N_{yy}^h, N_{xy}^h) \) developed under thermal load can be written as
\[
V(\delta) = \int_A \left\{ \frac{1}{2} N_{ax} \left( \frac{\partial \theta_x}{\partial x} \right)^2 + N_{aw} \left( \frac{\partial \theta_x}{\partial w} \right)^2 + 2N_{axw} \left( \frac{\partial \theta_x}{\partial x} \right) \left( \frac{\partial \theta_x}{\partial w} \right) + \frac{\mu}{2} \left( \frac{\partial \theta_x}{\partial x} \right)^2 + \frac{\mu}{2} \left( \frac{\partial \theta_x}{\partial w} \right)^2 \right\} \, dA
\]
\[
+ \int_A \left\{ N_{ay} \left( \frac{\partial \theta_y}{\partial y} \right)^2 + N_{aw} \left( \frac{\partial \theta_y}{\partial w} \right)^2 + 2N_{ayw} \left( \frac{\partial \theta_y}{\partial y} \right) \left( \frac{\partial \theta_y}{\partial w} \right) + \frac{\mu}{2} \left( \frac{\partial \theta_y}{\partial y} \right)^2 + \frac{\mu}{2} \left( \frac{\partial \theta_y}{\partial w} \right)^2 \right\} \, dA
\]
\[
+ 2N_{xy} \left( \frac{\partial \theta_x}{\partial y} \right) \left( \frac{\partial \theta_y}{\partial x} \right) + \frac{\mu}{2} \left( \frac{\partial \theta_x}{\partial y} \right)^2 + \frac{\mu}{2} \left( \frac{\partial \theta_y}{\partial x} \right)^2 \right\} \, dA
\tag{18}
\]

The work done by the applied non-conservative loads is
\[ W(\delta) = \int_A \Delta p \, w \, dA \]  

(19)

where \( \Delta p \) is the aerodynamic pressure. The aerodynamic pressure based on first-order, high Mach number approximation to the linear potential flow [15] is

\[ \Delta p = \frac{\rho_a U_a^2}{\sqrt{M_\infty^2 - 1}} \left[ \frac{\partial w}{\partial x} + \frac{1}{U_a} \frac{M_\infty^2 - 2}{M_\infty^2 - 1} \frac{\partial w}{\partial t} \right] \]  

(20)

where \( \rho_a, U_a \) and \( M_\infty \) are the free stream air density, velocity and Mach number, respectively. Further, it has been shown in Ref. [15] that the two-dimensional (2D) static aerodynamic approximation provides results that are in complete agreement with those based on exact aerodynamic theories for Mach numbers between \( \sqrt{2} \) and 2. The aerodynamic pressure for high supersonic speed, within the 2D static approximation [16, 17] is given as

\[ \Delta p = \frac{\rho_a U_a^2}{\sqrt{M_\infty^2 - 1}} \left[ \frac{\partial w}{\partial x} \right] \]  

(21)

Substituting the Eqs. (16) – (21) in Lagrange’s equation of the motion, one obtains the governing equation as [17]

\[ [M] \ddot{\delta} + \left[ K \right] + T_h \left[ K_G \right] + \lambda \left[ \bar{A} \right] \delta = 0 \]  

(22)

where \( \lambda = \frac{\rho_a U_a^2}{\sqrt{M_\infty^2 - 1}} \), \([M]\) and \([\bar{A}]\) are the consistent mass, and aero-dynamic force matrices, respectively. \([K]\) and \([K_G]\) are stiffness matrix, and geometric stiffness matrix per unit temperature rise, respectively. \(T_h\) is the temperature rise. Substituting the harmonic motion in the form \( \delta = \{ \delta_0 \} e^{i\omega t}\) in Eq. (22), the governing equation leads to

\[ \left[ \bar{K} - \bar{k} [I] \right] \{ \delta_0 \} = 0 \]  

(23)

where \( \bar{K} = \left[ \left[ K \right] + T_h \left[ K_G \right] + \lambda \left[ \bar{A} \right] \right]^{-1} [M] \) and \( \bar{k} \) is equal to \( \frac{1}{\omega^2} \). \([I]\) is the identity matrix.

The problem is now reduced to that of finding out the eigenvalues and corresponding mode shapes of the system for a given value of the aerodynamic pressure parameter, \( \lambda \). The eigenvalues can be obtained by making use of QR algorithm [18]. When \( \lambda = 0 \), the eigenvalue \( \omega \) is real and positive definite, since the stiffness matrices and mass matrix are symmetric and positive definite. However, the aerodynamic matrix \( \left[ \bar{A} \right] \) is not symmetric and hence complex eigenvalues \( \omega \) are expected for \( \lambda > 0 \). As \( \lambda \) increases monotonically from zero, two of these eigenvalues will approach each other, coalesce to \( \omega_c \) at \( \lambda = \lambda_c \), and become complex conjugate pairs, i.e., \( \omega = \omega_c \pm i\Gamma \), for \( \lambda > \lambda_c \). Here, \( \lambda_c \) is considered to be the value \( \lambda \) at which the first coalescence occurs.

**Results and Discussion**

Here, an eight-noded shear flexible quadrilateral plate element, based on field-consistency approach [12, 13] is extended to analyse the flutter characteristics pertaining to functionally graded plates. It has five nodal degrees of freedom associated to \( u_0, v_0, w, \theta_x \) and \( \theta_y \) at eight nodes in the element. The formulation includes transverse shear deformation, and in-plane and rotary inertia effects. The geometric stiffness is essentially a function of the in-plane stress distribution in the element due to applied temperature distribution over the plate. The simply supported boundary condition considered here is

\[ u_0 = w = \theta_y = 0 \quad \text{on} \quad x = 0, a \]
\[ v_0 = w = \theta_x = 0 \quad \text{on} \quad y = 0, b \]

Figure 2a shows the variation of the volume fractions of ceramic in the thickness direction \( z \) for the functionally graded plate. It can be noted here that the sum volume fractions of ceramic and metal is equal to unity. The FGM plate considered here consists of aluminum and alumina. The Young’s modulus, conductivity, and the coefficient of thermal expansion for alumina are
$E_c = 380 \text{ GPa}, K_c = 10.4 \text{ W/mK}, \alpha_c = 7.4 \times 10^{-6} \text{ (1/°C)}$
whereas for aluminum, these are $E_m = 70 \text{ GPa}$
$K_m = 204 \text{ W/mK}, \alpha_m = 23 \times 10^{-6} \text{ (1/°C)}$, respectively [14]. Poisson’s ratio is chosen as $\nu = 0.3$. Transverse shear coefficient $K_j$ is taken as 0.91. The temperature variation in the thickness direction as per Eq. (7) with the volume fraction index is presented in Fig. 2b for $T_r/T_m = 15$ and it can be noted that the temperature variation through the thickness of functionally graded plate is highly nonlinear compared to those of pure ceramic and metal cases ($k = 0$ and $k = 100$).

Before proceeding for the detailed analysis, the efficacy of the present formulation is tested for mesh convergence considering the flutter analysis of simply supported thin isotropic as well as functionally graded plates. Here, the non-dimensional flutter frequency

$$\bar{\omega}^2 = \omega^2 a^4 \left( \frac{\rho_m h}{D_m} \right) ; \rho_m \text{ is the mass density of metal and } D_m = \frac{E_m h^3}{12(1-\nu^2)} \text{ is the flexural rigidity of metal}$$

and the critical aerodynamic pressure $\lambda_c$, obtained based on progressive mesh refinement suggests that a 8x8 mesh is adequate to model the full plate as shown in Table-1. The results evaluated for isotropic case is found to be in very good agreement with the available results [19]. The coalescence of first two modes, and the change in the behaviour of flutter mode shape with the increase in dynamic pressure along the flow direction is highlighted in Fig. 3. It is observed from Fig. 3(b) that the location of maximum value of the mode shifts from centre of the plate to $x/h = 0.75$ as the dynamic pressure approaches the critical value.

Next, the flutter characteristics of square and rectangular plates ($a/h = 100$) made of aluminium-alumina (functionally graded material) is investigated, neglecting the influence of thermal load and is presented in Fig. 4. It is inferred from this Figure that, for a given thickness ratio, the critical non-dimensional aerodynamic pressure increases with the increase in the aspect ratio $a/b$. Also, it can be observed that, the critical non-dimensional aerodynamic pressure decreases with the increase in volume fraction index $k$. This is due to the fact that the stiffness is high for the ceramic panel and minimum for the metallic panel, and it degrades gradually with the increase in $k$. The effect of side-to-thickness ratio on critical non-dimensional

![](image)

Fig. 2 Variation of volume fraction of ceramic and temperature through thickness: (a) Volume fraction of ceramic; (b) Temperature

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Crit. pres.</th>
<th>Coales. freq.</th>
<th>$\bar{\omega}_c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2X2</td>
<td>252.15</td>
<td>1113.90</td>
<td></td>
</tr>
<tr>
<td>4X4</td>
<td>487.69</td>
<td>1772.37</td>
<td></td>
</tr>
<tr>
<td>6X6</td>
<td>510.16</td>
<td>1837.66</td>
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</tr>
<tr>
<td>8X8</td>
<td>511.11</td>
<td>1840.29</td>
<td></td>
</tr>
<tr>
<td>Ref. [19]</td>
<td>512.33</td>
<td>1846.55</td>
<td></td>
</tr>
</tbody>
</table>

The aerodynamic pressure of functionally graded plates is also examined assuming two values for $a/h (=20$ and $100)$. It can be noticed that the increase in thickness ratio ($a/h$) increases the critical non-dimensional flutter speed or dynamic pressure.

Functionally graded plates are generally used in high thermal gradient applications. Hence, solving the flutter problem in thermal environment is in order. Here, the constituent of functionally graded material is Silicon nitride and Stainless steel, referred as $Si_3N_x$/SUS304. The metal properties $P$, such as Young’s modulus and thermal coefficient of expansion of this material are assumed to be temperature dependent and can be
expressed as in Eq. (4). The temperature coefficients corresponding to Si3N4 / SUS304 are listed in Table-2 [20]. The mass density and thermal conductivity are: 

\[ \rho_c = 2370 \text{ kg/m}^3, K_c = 9.19 \text{ W/mK} \] for Si3N4; and 

\[ \rho_m = 8166 \text{ kg/m}^3, K_m = 12.04 \text{ W/mK} \] for SUS304.

Poisson’s ratio \( \nu \) is assumed to be a constant and equals to 0.28.

Finally, the flutter characteristics of functionally graded panels are investigated considering with and without thermal gradient. The material properties are evaluated at \( T = 300K \) for the uniform temperature case. For the chosen thickness ratio \( (a/h=20) \), the non-dimensional frequency \( \bar{\omega}^* = \omega^* \frac{a^3}{D_{m0}} \) \( (\rho_{m0}) \) and \( D_{m0} \) belong to metal at \( T_m = 300K \) and critical aerodynamic pressure evaluated for various aspect ratios are tabulated in Table-3. The influence of temperature gradient \( (T_e = 600K \text{ and } T_m = 300K) \) on the flutter behavior is examined considering the appropriate temperature for the evaluation material properties. The temperature field is assumed to vary only in the thickness direction and determined by the expression given in Eq. (7). The results obtained are also given in Table-3. As expected, the critical pressures and coalescence frequencies decreases under thermal environment. It can be noted here that all the coalescence modes are one and two for the cases \( a/b = 1 \) and 2. For the higher aspect ratio considered here \( (a/b = 5) \), the coalescence occurs at higher modes.

**Conclusions**

The supersonic flutter of functionally graded flat panels has been analyzed based on first-order shear deformation theory through finite element approach. The material properties are assumed to be varied through the thickness direction based on power law distribution. The aerodynamic force is accounted for assuming the first-order high Mach number approximation to linear potential flow theory. The effectiveness of volume fraction index, aspect and thickness ratios, and thermal environment on the critical aerodynamic pressure has been demonstrated. From the detailed parametric study, the following observations can be made:
With the increase in volume fraction index, critical aerodynamic pressure decreases due to the degradation of stiffness.

The critical aerodynamic pressure increases with the increase in the aspect ratio and the rate of increase depends on volume fraction index.

The increase in thickness ratio of the plate increases the critical non-dimensional flutter speed and it depends on the value of volume fraction index.

Thermal gradient decreases the critical flutter speed, as expected.

Coalescence of higher modes is possible due to the existence of extensional-bending coupling with volume fraction index, depending on aspect ratio and thermal gradient.

References

Table 2. Temperature dependent coefficients for material Si₃N₄ / SUS304, Ref. [20].

<table>
<thead>
<tr>
<th>Materials</th>
<th>Properties</th>
<th>P₀</th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
<th>P₄</th>
<th>P₅</th>
</tr>
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<tbody>
<tr>
<td>Si₃N₄</td>
<td>E (Pa)</td>
<td>348.43e+9</td>
<td>0.0</td>
<td>-3.070e-4</td>
<td>2.160e-7</td>
<td>-8.946e-11</td>
<td>322.2715e+9</td>
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<td>α (1/K)</td>
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<td>9.095e-4</td>
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<tr>
<td>SUS304</td>
<td>E (Pa)</td>
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<td>207.7877e+9</td>
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<tr>
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<td>α (1/K)</td>
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<td>8.086e-4</td>
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<td>0.0</td>
<td>15.321e-6</td>
</tr>
</tbody>
</table>

Table 3. Flutter behaviour of temperature dependent FGM plate (Material : Si₃N₄ / SUS304, a/h=20, simply supported).

<table>
<thead>
<tr>
<th>a/h</th>
<th>k</th>
<th>Tₑ = 300 , T_m = 300</th>
<th>Coalescence</th>
<th></th>
<th>Tₑ = 600 , T_m = 300</th>
<th>Coalescence</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>In-vacuo</td>
<td></td>
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<td>In-vacuo</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ω₁²*</td>
<td>ω₂²*</td>
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* Coalescence modes are 2&3, Corresponding ω₁² and ω₂² represent ω₁² and ω₂²
** Coalescence modes are 3&4, Corresponding ω₁² and ω₂² represent ω₁² and ω₂²


