EFFECT OF MATERIAL DAMPING CHARACTERISTICS ON DYNAMIC ANALYSIS OF ROTOR-BEARING SYSTEMS

B.B. Maharathi*, S.N. Mishra**, P. Acharya* K.C. Singh* and R. Das*

Abstract

Dynamical behaviour of rotor-bearing systems is significantly influenced by a large number of parameters related to rotor, disk and bearing. However, inherent material damping, parameters of rotor play an important role on dynamic characteristics of rotor-bearing systems. Conventional transfer matrix method is modified by incorporating viscous and hysteretic form of internal damping parameters of rotor in the analysis. A three-disk rotor system mounted on rigid supports as well as on flexible bearings is taken as a physical model to demonstrate the effects of damping on dynamic characteristics of the system. The viscous damping of the rotor system diminishes the dynamic response where as the hysteretic form of damping always destabilises the system.

Notation

\( A \) = cross-sectional area of shaft

\( a \) = major axis of elliptical whirling orbit

\( [B] \) = bearing matrix

\( b \) = minor axis of elliptical whirling orbit

\( C \) = radial clearance of bearing

\( C_{yy}, C_{zz} \) = direct damping coefficients of bearing

\( C_{yz}, C_{zy} \) = cross damping coefficients of bearing

\( C \) = damping parameter

\( C_c \) = critical damping parameter

\( [D] \) = modified disk matrix

\( D \) = disk diameter

\( D_b \) = bearing diameter

\( d \) = shaft diameter

\( E \) = Young's modulus of shaft material

\( e \) = eccentricity

\( [F] \) = field matrix of rotor segment

\( [F] \) = modified field matrix

\( G \) = shear modulus

\( l \) = length of the shaft segment

\( l_p, l_T \) = polar and lateral mass moment of inertia of disk, respectively

\( K \) = shaft stiffness

\( K_{yy}, K_{zz} \) = direct stiffness coefficients of bearing

\( K_{yz}, K_{zy} \) = cross stiffness coefficients of bearing

\( L_y \) = total length of the shaft

\( L_y \) = length of bearing

\( m \) = effective mass of the shaft segment per unit length

\( m_b \) = mass at the supported bearing

\( m_e \) = mass of the unbalance

\( m_d \) = mass of the disk

\( m_i \) = mass of the shaft and disks

\( m_s \) = mass of the shaft

\( N \) = rotation of shaft (rpm)

\( M_y, M_z \) = moments about Y and Z axis, respectively

\( [P] \) = point matrix of disk

\( [P] \) = modified point matrix of disk

\( r_w \) = radius of the elliptical whirling orbit

\( \{S_y\}, \{S_z\} \) = state vectors in horizontal \((X-Y)\) and vertical \((X-Z)\) planes, respectively

\( \{S\} \) = modified state vector

\( S \) = summerfield number

\( [U] \) = overall transfer matrix

\( U \) = unbalance force \((=m_e \times e)\)

\( U_y, U_z \) = components of unbalance force along Z and Y axis respectively

\( \nu, w \) = deflections along Y and Z axis, respectively

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V_y, V_z = shear forces along Y and Z axis, respectively
W = bearing static load
α = shear deformation factor (= 0.75)
ε, β = co-ordinates of unbalance mass (radial and angular coordinates respectively)
γ = material property
θ, φ = slopes in vertical (X-Z) and horizontal (X-Y) plane, respectively
ζ = viscous damping factor (= C/C_μ)
ω = rotating speed (rad/sec)
ω_r = rigid bearing critical speed (rad/sec)
Ω = whirling speed (rad/sec)
μ = coefficient of viscosity

Subscripts
\( c, s \) = used for cosine and sine terms
\( \alpha, \beta \) = stage numbers

Superscripts
\( L, R \) = left and right of the station, respectively
\( t \) = transpose of a matrix

Introduction

On account of ever-increasing demand for transmitting power and maintaining high speed, the rotor-bearing systems employed in various industrial machines such as, steam and gas turbines, turbogenerators, internal combustion engines, reciprocating and centrifugal compressors etc. need careful design for their operations. In view of this, dynamic analysis of such rotor-bearing systems playa major role for their safe design. The dynamic behaviour of such rotors is generally influenced by the unbalance mass of the system and fluid-film bearing coefficients.

Methods for evaluation of dynamic response and critical speeds are based on either Holzer-Myklestad-Prohl model or finite element method. However, transfer matrix method (TMM) is commonly used for dynamic analysis of rotor-bearing systems in the frequency domain incorporating various influencing parameters related to rotor, disk and fluid-film bearings. The conventional transfer matrix method is first applied to rotor-bearing systems by Prohl [1], Kikuchi [2] and Lund [3-4] considering different aspects of rotor-disk systems mounted on rigid bearings as well as on isotropic bearings. Maharathi and Behera [5-10] made significant modifications of transfer matrix method based on lumped and continuous system model considering various parameters such as unbalance mass of rotor/disk, gyroscopic moments of rotor/disk shear deformation, rotary inertia, axial force and tangential torque acting on rotor and dynamic coefficients of fluid-film bearings with its rotational stiffness, damping and foundation characteristics.

The effects of viscous and structural damping on the dynamics of rotors, supported on rigid bearings have been investigated by many researchers [11-16] using classical methods based on partial differential equations. In the present investigation, conventional transfer matrix method based on lumped system model has been modified incorporating viscous and hysteretic damping parameter for dynamic analysis of rotor bearing systems to fit the synchronous elliptical orbits of the systems. The conventional transfer matrix method is modified to the order of 16 x 16 for evaluation of dynamic response and identification of critical speed at the maximum amplitude of vibration of the systems within the operational speed range and to demonstrate the significant effects of damping parameters on dynamic characteristics.

Transfer Matrix Analysis

Figures 1 represent the mathematical models of rotor-disk systems supported on flexible fluid-film bearings. These rotor systems consist of a number of point masses which are separated by massless shaft segments called fields. The state vector at any station is based on elements constituting the variables such as deflection, slope, bending moment and shear force in both horizontal (X-Y) and vertical (X-Z) planes. Due to the presence of unbalance in rotor-disk systems, the excitation forces are expressed in cosine and sine terms, so the state vector quantities \( w, \theta, M_y, V_z \) in X-Z plane and \( v, \phi, M_z, V_y \) in X-Y plane can be expressed in terms of cosine and sine terms to fit the elliptical orbit under synchronous whirling condition \( (\Omega=\omega)\). They are as follows:

For vertical (X-Z) plane:

\[
\begin{align*}
w &= w_c \cos \Omega t + w_s \sin \Omega t \\
\theta &= \theta_c \cos \Omega t + \theta_s \sin \Omega t \\
M_y &= M_{yc} \cos \Omega t + M_{ys} \sin \Omega t \\
V_z &= V_{zc} \cos \Omega t + V_{zs} \sin \Omega t
\end{align*}
\]

(1a)
For horizontal (X-Y) plane:

\[ v = v_c \cos \Omega t + v_y \sin \Omega t \]
\[ \phi = \phi_c \cos \Omega t + \phi_y \sin \Omega t \]
\[ M_z = M_{cz} \cos \Omega t + M_{zy} \sin \Omega t \]
\[ V_y = V_{yc} \cos \Omega t + V_{zy} \sin \Omega t \]

(1b)

Referring to Fig. 2, the radius vector of the whirl orbit is

\[ r_w = \sqrt{w_c^2 + v_y^2} \]

(2)

From equation (1), we get

\[ r_w = \left( (w_c \cos \Omega t + w_y \sin \Omega t)^2 + (v_c \cos \Omega t + v_y \cos \Omega t)^2 \right)^{\frac{1}{2}} \]

(3)

The above equation can be written as

\[ r_w = \left[ \frac{1}{2} \left( w_c^2 + w_y^2 + v_c^2 + v_y^2 + (w_c^2 - w_y^2 + v_c^2 - v_y^2) \cos 2\Omega t + 2(w_c w_y + v_c v_y) \sin 2\Omega t \right) \right]^{\frac{1}{2}} \]

(4)

the radius vector is maximum or minimum when

\[ \frac{d}{d(\Omega t)} \left[ \left( w_c^2 - w_y^2 + v_c^2 - v_y^2 \right) \cos 2\Omega t + 2(w_c w_y + v_c v_y) \sin 2\Omega t \right] = 0 \]

(5)

i.e.,

\[ \tan 2\Omega t = \frac{2(w_c w_y + v_c v_y)}{w_c^2 - v_c^2 - w_y^2 - v_y^2} \]

(6)

Using equation (6) in (4), the major and minor axis of the elliptical orbit shown in Fig. 2 are,

\[ a, b = \left[ \frac{1}{2} \left( w_c^2 + w_y^2 + v_c^2 + v_y^2 \pm (w_c^2 - w_y^2 + v_c^2 - v_y^2) \cos \theta \right) + 4(w_c w_y + v_c v_y) \right]^{\frac{1}{2}} \]

(7)

For systems with isotropic bearings, the elliptical orbit becomes circular, since \( w_c = -v_y \) and \( w_y = v_c \) and the above equation becomes

\[ a = b = \sqrt{w_c^2 + v_y^2} \]

(8)

Transfer Matrix Analysis of Rotor

From the elementary theory of elasticity as shown in Figure 3, the transfer equations for a shaft section can be written as follows:

In the X-Z plane:

\[ -w_i^L = -w_{i-1}^R + \theta_{i-1}^R l_i + M_{y,i-1}^R \frac{l_i^2}{(2EI)_i} + V_{z,i-1}^L \frac{l_i^3}{(6EI)_i} \]

\[ \theta_i^L = \theta_{i-1}^R + M_{y,i-1}^R \frac{l_i}{(EI)_i} + V_{z,i-1}^L \frac{l_i^2}{2(6EI)_i} \]

\[ M_{L,i}^L = M_{y,i-1}^L + V_{z,i-1}^L l_i \]

\[ V_{zi}^L = V_{z,i-1}^L \]

(9a)

Similarly in the X-Y plane:
EFFECT OF MATERIAL DAMPING CHARACTERISTICS

where \( \{S\}_{J_{17x17}} \) and \( \{F\}_{J_{17x17}} \) are modified state vectors and field matrix of the rotor segment respectively,

\[
\begin{bmatrix}
[F] & [0] & [0] & [0] & [0] \\
[0] & [F] & [0] & [0] & [0] \\
[0] & [0] & [F] & [0] & [0] \\
[0] & [0] & [0] & [F] & [0] \\
[0] & [0] & [0] & [0] & [1]
\end{bmatrix}
\]

(11)

and

\[
\{s\} = \begin{bmatrix}
-w_c, \theta_c, M_{yc}, V_{zc}, -w_y, \theta_c, M_{yy}, V_{zz}, v_c, \phi_c, M_{zc}, \\
-V_{yc}, v_x, \phi_x, M_{xz}, -V_{yz}, y_c \end{bmatrix}_{J_{17x1}}
\]

(12)

\([F]\) is a field matrix of order 4x4, \( [0] \) is a null matrix of order 4x4 and \( 't' \) is the transpose of the array. For the purpose of response calculations, the identity \( I = 1 \), is added to 16 equations of the total state vector as indicated in equations (10),(11) and (12).

**Modified Transfer Matrix Analysis of Rotor**

The rotor sections are assumed to have internal material damping of hysteretic type where the dissipated energy is independent of frequency. The stress leads the corresponding strain by an angle \( \gamma \), which is a material property. The hysteresis loop in the stress-strain plane is an ellipse whose area, proportional to \( \sin \gamma \), gives measure of the dissipated energy such that

\[
\sin \gamma = \frac{\epsilon}{\sqrt{1+\epsilon^2}}
\]

(13)

Where \( \pi \epsilon \) is equal to the logarithmic decrement for the rotor. The relationship between bending strain and bending stress has been established by Timoshenko [17] and are as follows:

\[
\frac{d\theta}{dx} = \frac{1}{EI} M_{\gamma} \cos \gamma - \frac{1}{EI} M_{\gamma} \sin \gamma
\]

(14a)

and

\[
\frac{d\phi}{dx} = \frac{1}{EI} M_{\gamma} \sin \gamma - \frac{1}{EI} M_{\gamma} \cos \gamma
\]

(14b)

or
The hysteretic form of internal damping, used in the following is believed to be more valid representation for a practical rotor system than viscous damping, but the subject matter is not well understood.

By incorporating the shear factor and the hysteretic form of damping in the transfer equations of the shaft, equations (9) are modified and as follows:

In the X-Z plane,

\[
\frac{d\theta}{dx} = \frac{1}{EI\sqrt{1+\varepsilon^2}} (M_y - e M_z) \tag{15a}
\]

and

\[
\frac{d\phi}{dx} = \frac{1}{EI\sqrt{1+\varepsilon^2}} (M_z - e M_y) \tag{15a}
\]

Substitution of equations (1a) in (16a) and (lb) in (16b) yields the modified field matrix and state vectors of the rotor segment vibrating in Z-X and X- Y planes.

Point Matrix of Concentrated Mass

Consider a n mass system, each mass representing either a gear, a disk or a flywheel etc. as shown in Fig.1. All these masses are taken as lumped one. Fig.4 gives the shear force equilibrium relations of the concentrated mass at the station considering inertia force, damping and exciting forces due to the presence of unbalance mass in the rotor/disk. They are as follows:

In the X-Z plane;

\[
M_{v, z} = \dot{M}_{v, z} + \dot{V}_{v, z} + 2m_{v} \omega_{y} \dot{V}_{v, z} - \omega^2 U_{v, z} \cos \omega t
\]

\[
+ \omega^2 U_{v, z} \sin \omega t \tag{17a}
\]

Similarly in the X- Y plane;

\[
M_{v, y} = \dot{M}_{v, y} + \dot{V}_{v, y} + 2m_{v} \omega_{y} \dot{V}_{v, y} - \omega^2 U_{v, y} \cos \omega t
\]

\[
- \omega^2 U_{v, y} \sin \omega t \tag{17b}
\]
The deflection, slope and moment are continuous at the \( t^n \) position. For synchronous whirling, whirling speed becomes equal to rotation speed \((\Omega = \omega)\). \( \omega_{cr} \) represents for critical speed of rotor supported on rigid bearing and \( \xi \) represents for viscous damping parameter.

\[
\omega_{cr} = \sqrt{\frac{K}{M_t}} \tag{18}
\]

where \( K \) and \( M_t \) represents for shaft stiffness and total mass (mass of the shaft and masses of all mountings), respectively.

The rigid bearing critical speed of the rotor system can be found out by using the conventional energy principle such as; Dunkerleys method or Rayleighs method. They are as follows:

**Dunkerleys Method**

\[
\frac{1}{\omega_{cr}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \cdots + \frac{1}{\omega_n^2} \tag{19}
\]

where \( \omega_1, \omega_2, \omega_3, \ldots, \omega_n \) are the critical speeds of the system carrying individual components and \( \omega_{cr} \) represents the first critical speed of the complete system.

**Rayleighs Method**

\[
\omega_{cr}^2 = \frac{\sum_{i=1}^{n} M_i y_i^2}{\sum_{i=1}^{n} M_i y_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} M_i M_j \sin^2 \xi_{ij} \omega^2} \tag{20}
\]

where \( M_1, M_2, M_3, \ldots, M_n \) are the masses of different components mounted on the rotor and \( y_1, y_2, y_3, \ldots, y_n \) are the static deflections of the rotor at the locations of these components.

Substitution of equation (1) in (17) yields the modified point matrix \([\bar{P}]\) at the \( t^n \) position.

\[
[\bar{S}]_{ij(17,1)} = \sum_{i=1}^{L} \sum_{j=1}^{L} [\bar{P}]_{ij(17,1)} \{S\} \tag{21}
\]

The non-zero elements of \([\bar{P}]\) are as follows:

\[
[\bar{P}]_{i,j} = 1, i = 1, 2, 3, \ldots, 17
\]

\[
\bar{P}_{4,1} = \bar{P}_{8,5} = \bar{P}_{12,9} = \bar{P}_{16,13} = m\omega^2
\]

\[
\bar{P}_{4,5} = \bar{P}_{6,1} = 2m\xi\omega, \omega
\]

\[
\bar{P}_{4,17} = \bar{P}_{16,17} = -U_3\omega^2
\]

\[
\bar{P}_{12,13} = \bar{P}_{16,9} = -2m\xi\omega_{cr}, \omega
\]

\[
\bar{P}_{8,17} = \bar{P}_{12,17} = U_1\omega^2 \tag{22}
\]
Transfer Matrix Analysis for Bearings

Since the bearings are part and parcel of a rotating system, they play a major role for the dynamic behaviour of rotor. These bearings may be considered either rigid or flexible depending on the accuracy required in the analysis. However, fluid-film bearings are to be considered as flexible because of their stiffness and damping effects. These coefficients of such bearings significantly alter the dynamic characteristics of the rotor.

Analysis for Rigid Bearing

For the evaluation of dynamic characteristics of rotor-disk system mounted on rigid bearings, as shown in Fig.5, the transfer matrix of such system can be written as:

\[
\begin{align*}
\{\dot{S}\}_l &= [A]_0 \{S\}_0 \\
\{\dot{S}\}_m &= [B]_0 \{S\}_0 \\
\{\dot{S}\}_n &= [C]_0 \{S\}_0
\end{align*}
\]

where \(\{S\}\) is the state vector of 17 x 1 in size, \([A]_0\), \([B]_0\) and \([C]_0\) are overall transfer matrices for the left overhang, middle span and right overhang of the rotor respectively. \(\{S\}_0\) is the state vector at the left free end.

Analysis for Flexible Bearing

In the rotor system, the fluid-film bearings play an important role for the dynamic behaviour analysis. Since the squeezed thin film between the journal and the ring provides the effects of spring and damping as shown in Fig.6, the asymmetry stiffness and damping coefficients of bearings dominate the dynamic response, critical speed and threshold speed of instability. These asymmetric stiffness and damping coefficients of fluid-film bearings are derived from the steady state condition of the journal as shown in Fig.7. When the journal is displaced by \(\Delta z\) and \(\Delta y\) from the equilibrium position, then the vertical and horizontal components of loads can be written down using Taylor's series from equilibrium position as follows:
\[ W_z = W_{z0} + W_{z,r} \Delta z + W_{z,y} \Delta y + W_{z,t} \Delta z + W_{z,y} \Delta y \]
\[ W_y = W_{y0} + W_{y,r} \Delta z + W_{y,z} \Delta y + W_{y,t} \Delta z + W_{y,y} \Delta y \]  
(25a)

and the above equations can be written as:
\[ W_z = W_{z0} + K_{z,r} \Delta z + K_{z,y} \Delta y + C_{z,r} \Delta z + C_{z,y} \Delta y \]
\[ W_y = W_{y0} + K_{y,r} \Delta z + K_{y,y} \Delta y + C_{y,r} \Delta z + C_{y,y} \Delta y \]  
(25b)

where
\[ K_{zz} = \left( \frac{\partial W_z}{\partial z} \right) \quad K_{zy} = \left( \frac{\partial W_z}{\partial y} \right) \]
\[ K_{yz} = \left( \frac{\partial W_y}{\partial z} \right) \quad K_{yy} = \left( \frac{\partial W_y}{\partial y} \right) \]
\[ C_{zz} = \left( \frac{\partial W_z}{\partial z} \right) \quad C_{zy} = \left( \frac{\partial W_z}{\partial y} \right) \]
\[ C_{yz} = \left( \frac{\partial W_y}{\partial z} \right) \quad C_{yy} = \left( \frac{\partial W_y}{\partial y} \right) \]  
(26)

In the above, \( K_{ij} \) are stiffness coefficients, with \( i \) representing the direction of force and \( j \) representing the direction of displacement. Under dynamic conditions, the journal centre will have velocities \( \dot{z}, \dot{y} \) and they induce additional force \( W_z, W_y \) due to squeeze film effect. These forces are denoted with coefficients \( C_{zz}, C_{zy}, C_{yz}, C_{yy} \) to define the total changes in forces \( W_z, W_y \). These coefficients are damping coefficients, since the squeeze film force is taken proportional to the velocities. These eight linear bearing coefficients \( (K_{zz}, K_{zy}, K_{yz}, C_{zz}, C_{zy}, C_{yz}, C_{yy}) \) are derived by treating the fluid-film as a laminar flow and are governed by Reynold's equation \([18,19]\). The shear force relations across a bearing station as shown in Fig. 8 are as follows:

In the X-Z plane:
\[ V_{z,i} = V_{z,i}^L + K_{z,y} v_{z,i} + K_{z,z} v_{z,i} + C_{z,y} \dot{v}_{z,i} + C_{z,z} \dot{v}_{z,i} + m_y \ddot{v}_{z,i} \]  
(27a)

similarly in the X-Y plane:
\[ V_{y,i} = V_{y,i}^L + K_{y,y} v_{y,i} + K_{y,z} v_{y,i} + C_{y,y} \dot{v}_{y,i} + C_{y,z} \dot{v}_{y,i} + m_y \ddot{v}_{y,i} \]  
(27b)

where \( m_y \) is the mass of the shaft at the supported bearing.

The deflection, slope and moment quantities are continuous at the bearing location. Substitution of equation (1) in (27) yields the bearing transfer matrix, which can be written as follows:
\[ \{ \ddot{S} \}_{3 \times 1}^R = [B]\{ S \}_{3 \times 1}^L \]  
(28)

The non-zero entries in \([B]\) are as follows:
The stiffness and damping coefficients of plain-cylindrical journal bearing ($L_b/D_b=0.5$) as used in the numerical analysis can be taken from Figs. 9 and 10. These figures can be expressed as polynomial functions of Sommerfield number ($S$) for obtaining the stiffness and damping coefficients [5-10].
where
\[ S = \frac{\mu D_b P N}{W} \times \left( \frac{R_0}{C} \right) \] (31)

Overall Transfer Matrix

The overall transfer matrix of whole rotor-disks systems mounted on fluid film bearings can be represented by
\[ \{ S \}_{b(17x1)} = \left[ \bar{U} \right]_{b(17x1)} \left[ S \right]_{b(17x1)} \] (32)

where
\[ \left[ \bar{U} \right] = \left[ \bar{F} \right]_a \left[ B \right] \left[ \bar{F} \right]_a \left[ D \right] \ldots \left[ \bar{F} \right]_b \left[ D \right] \ldots \left[ \bar{F} \right]_l \] (33)

Numerical Analysis

A simplified rotor-disks system mounted on two plain cylindrical journal bearings as shown in Fig.11 is taken as the physical model. Details of the rotor system are presented in Table-1. For evaluation of the dynamic characteristics of the physical rotor model, it is quite essential to obtain the overall transfer matrix of the given model. Equation (30) is to be solved in order to obtain \( \{ S \}_{b(17x1)} \) at the left free end and then the state vector at the unbalanced disk position (at the middle disk) are evaluated through matrix 'operations in order to find dynamic response and identification of critical speed of the given model. The dynamic characteristic curves are presented in Figs.12 to 17 considering viscous damping and hysteretic form of damping parameters of rotor, mounted on rigid bearings as well as on fluid-film bearings respectively. Numerical results of the above system mounted on rigid and fluid bearings under different damping conditions are presented in Table-2 and 3.

<table>
<thead>
<tr>
<th>Table-1: Details of Three-disk Rotor System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Details of Rotor</strong></td>
</tr>
<tr>
<td>i) Young’s modulus (E)</td>
</tr>
<tr>
<td>ii) Shear modulus (G)</td>
</tr>
<tr>
<td>iii) Shape factor (α)</td>
</tr>
<tr>
<td>iv) Diameter (d_a)</td>
</tr>
<tr>
<td>v) Length (L_a)</td>
</tr>
<tr>
<td>vi) Mass density (ρ)</td>
</tr>
<tr>
<td><strong>Details of disk</strong></td>
</tr>
<tr>
<td>i) Mass of each disk (M_a)</td>
</tr>
<tr>
<td>ii) Diameter of each disk (d_a)</td>
</tr>
<tr>
<td>iii) Thickness of each disk (h)</td>
</tr>
<tr>
<td>iv) Unbalance at the middle disk</td>
</tr>
<tr>
<td><strong>Details of bearing</strong></td>
</tr>
<tr>
<td>i) Length (L_b)</td>
</tr>
<tr>
<td>ii) Diameter (D_b)</td>
</tr>
<tr>
<td>iii) Clearance (C)</td>
</tr>
<tr>
<td>iv) Viscosity (μ)</td>
</tr>
</tbody>
</table>
Table-2: Numerical Results of Dynamic Response and Critical Speed of Rotor System Considering Viscous Damping Parameter

<table>
<thead>
<tr>
<th>Bearing Condition</th>
<th>Rotor Parameter</th>
<th>Maximum Dynamic Response (micron)</th>
<th>Critical Speeds (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid bearing</td>
<td>$\xi = 0$</td>
<td>Infinite</td>
<td>2600, 2800</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.1$</td>
<td>100.57</td>
<td>2600, 2800</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.2$</td>
<td>48.68</td>
<td>2600, 2800</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.3$</td>
<td>34.98</td>
<td>2600, 2800</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.4$</td>
<td>29.44</td>
<td>2600, 2800</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.5$</td>
<td>24.39</td>
<td>2600, 2800</td>
</tr>
<tr>
<td>Fluid-film bearing</td>
<td>$\xi = 0.1$</td>
<td>21.56</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.2$</td>
<td>15.09</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.3$</td>
<td>12.23</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.4$</td>
<td>10.59</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>$\xi = 0.5$</td>
<td>9.5</td>
<td>2600</td>
</tr>
</tbody>
</table>

Table-3: Numerical Results of Dynamic Response and Critical Speed of Rotor System Considering Hysteretic Damping Parameter

<table>
<thead>
<tr>
<th>Bearing Condition</th>
<th>Decay Factor of rotor ($\epsilon$)</th>
<th>Maximum Dynamic Response (micron)</th>
<th>Critical Speeds (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid-film bearing</td>
<td>$\epsilon = 0.0$</td>
<td>70.51</td>
<td>2640</td>
</tr>
<tr>
<td></td>
<td>$\epsilon = 0.1$</td>
<td>107.81</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>$\epsilon = 0.3$</td>
<td>113.00</td>
<td>2500</td>
</tr>
<tr>
<td></td>
<td>$\epsilon = 0.5$</td>
<td>(i) 160.41</td>
<td>(i) 2400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(ii) 149.59</td>
<td>(ii) 2600</td>
</tr>
</tbody>
</table>

Discussion

The numerical results of dynamic characteristics of the given rotor-bearing system are presented in Figs.12 to 17 and they are described as follows:

The dynamic characteristics of rotor-disk system mounted on rigid-bearing neglecting internal damping parameter ($\xi = 0$) are shown in Fig.12 and it is seen that two critical speeds (2600 rpm and 2800 rpm) are observed within the operational speed range 1000 to 6000 rpm. The amplitude of vibration is infinite at the critical speed of 2600 rpm. Furthermore, the variation of dynamic response of the same system under different damping conditions ($\xi = 0.1$ to 0.5) is presented in Fig.13. From this figure, it is seen that the maximum dynamic response of the same system is reduced from infinite to 24.39 micron without affecting any change in critical speeds.

Figure 14 represents the response curves of the given rotor-disk system mounted on fluid-film bearings considering its asymmetric stiffness and damping coefficients ($K_{zz}, K_{yy}, K_{yz}, C_{zz}, C_{yy}, C_{yz}$) and damping parameters ($\xi = 0.1$ to 0.5) of rotor neglecting...
gyroscopic effects of the rigid disk and treating them as point mass. From this figure, it is seen that the second critical speed at 2800 rpm is disappeared and only one critical speed is occurred at 2600 rpm under different damping parametric conditions ($\xi = 0.1$ to $0.5$). However, due to the significant effect of bearing damping and rotor damping coefficients, the maximum dynamic response of such system reduces from 21.56 micron to 9.5 micron as shown in Table-2 and forward synchronous whirling action is observed ($\Omega = \omega$).

Figure 15 shows the dynamic characteristics of such rotor-disks-bearing system considering mass effect of the rotor, gyroscopic moments of the rigid disks and bearing parameters neglecting viscous damping parameter ($\xi = 0$). From the comparison of the Figs.12 and 15, it is observed that due to gyroscopic effects of the disk, critical speed is seen at 2640 rpm but the dynamic response is reduced from infinite to 70.51 micron possessing forward synchronous whirling action ($\Omega = \omega$).

Significant effects of hysteretic form of damping parameter of rotor mounted on fluid-film bearings with disk parameters are shown in Figs.16 and 17. From these figures, it is observed that the critical speed decreases from 2640 to 2400 rpm under different decay factor of rotor ($\epsilon = 0.1$ to $0.5$) but the response of the system increases from 70.50 micron to 160.41 micron. Sometimes two resonant peaks may occur when the rotor system possess backward whirling action as observed in Fig.17 at $\epsilon = 0.5$ and the rotor has a forward synchronous whirl when the speed of the rotor is either below the first critical speed (2400 rpm) or above the second critical speed (2600 rpm). However, when the rotor is rotating with a speed between these two criticals, there is a backward synchronous whirl of the rotor ($\Omega = \omega$).

Conclusions

From the above discussions, the following conclusions are made and they are as follows:

Viscous damping parameter of the rotor diminishes the dynamic response of the system.

Stiffness and damping coefficients of the fluid-film bearings significantly reduce the dynamic response without affecting critical speed of the system.

Hysteretic form of damping of the rotor always destabilises the system.

Transfer matrix method is a suitable technique for steady-state response analysis of rotor-disks-bearings system incorporating various influencing parameters in the theoretical model.

References


