A SIMPLE ENERGY METHOD TO PREDICT THE THERMAL POST BUCKLING BEHAVIOUR OF COLUMNS

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Introduction

Post buckling problem of uniform columns, subjected to either mechanical loads [1,2] or thermal loads [3] is essentially a complex nonlinear problem. In the case of the thermal post buckling of columns, with the ends immovable axially, both the versatile finite element method and the Rayleigh-Ritz method are successfully employed [3], through rigorous mathematical treatment. This complex nonlinear problem can be simply treated as an equivalent linear problem by suitably defining the axial compressive load developed due to the temperature rise from the stress free condition of the column. The simple energy method can then be used to obtain the thermal post buckling behaviour of columns.

It may be noted here that in the classical method it is necessary to assume displacement distributions for the lateral displacement w and the axial displacement u, and 'u' displacement distribution has to be compatible with 'w'. Even though simple displacement distributions for 'u' can be assumed for uniform columns with classical boundary conditions, it becomes virtually impossible to assume 'u' for other column configurations (like tapered). The present simple energy method does not require the assumption of 'u' and hence eliminates the complication in assuming a compatible 'u' distribution.

In the present note the simple energy method, after redefining the compressive load generated in the column, is demonstrated by predicting the thermal post buckling behaviour of simply supported and clamped uniform columns.

Energy Method

For a uniform column, subjected to a thermal load, undergoing large deflections, the strain energy U and the work done W by the induced compressive load are given by

\[ U = \frac{EI}{2} \int_{0}^{L} (w)^{2} \, dx \]  

and

\[ W = \frac{P_{eq}}{2} \int_{0}^{L} (w)^{2} \, dx \]

where, E is the Young's modulus, I is the area moment of inertia, L is the length of the column, w is the lateral displacement, x is the axial coordinate and ( )' represents differentiation with respect to the axial coordinate x.

In Eq. (2), \( P_{eq} \) is the effective mechanical equivalent of the thermal load, which is equal to

\[ P_{eq} = P_{NL} \cdot T \]

where, \( P_{NL} \) is the compressive load developed because of the temperature rise, beyond the critical temperature at which the column buckles, from the stress free temperature and T is the tension developed in the column due to large deflections. The expression for T is given by [4]

\[ T = \frac{EI}{2Lr^{2}} \int_{0}^{L} (w)^{2} \, dx \]

where, r is the radius of gyration.

Equation (3) can be written in non-dimensional form as,

\[ \lambda_{eq} = \lambda_{NL} - \lambda_{T} \]

where,

\[ \lambda_{eq} = \frac{P_{eq} L^{2}}{EI} \]

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\[ \lambda_{NL} = \frac{P_{NL} L^2}{EI} \]  
and  
\[ \lambda_T = \frac{T L^2}{EI} \]  
(7)  
(8)  

The thermal equivalent of \( \lambda_{NL} \) is given by  
\[ \lambda_{NL} = \frac{(\alpha \gamma_{NL}) L^2}{t} \]  
where, \( \alpha \) is the coefficient of linear thermal expansion, \( t \) is the temperature rise from the stress free state of the column and the subscript \( NL \) denotes nonlinear (or post buckling) value.

The present method is applicable to the thermal post buckling of columns, where the ends are axially immovable and hence a compressive load develops in the column. In the case of plates, the in plane displacements have to be constrained in the direction normal to the edges to obtain the effective compressive load.

By assuming suitable admissible functions, which satisfy the kinematic boundary conditions, of the column, for \( w \) as  
\[ w = a \cdot F(x) \]  
(10)

where, \( a \) is the central deflection of the column. The strain energy \( U \) and the work done \( W \) can be obtained. Equating \( U \) and \( W \), we get the value \( \lambda_{eq} \) as \( \beta \).

From Eq. (5)  
\[ \lambda_{NL} - \lambda_T = \beta \]  
(11)

When the column is just at the critical thermal load, the value of \( \lambda_T \) is equal to zero, as the column will not have any lateral deflection. Then Eq. (11) can be written as  
\[ \lambda_L = \beta \]  
(12)

where, \( \lambda_L \) is the linear critical thermal load parameter given by

\[ \lambda_L = \frac{(\alpha \gamma_L) L^2}{t} \]  
(13)

where, the subscript \( L \) denotes linear (or buckling) value.

From Eqs. (11) and (12), the expression for \( \frac{\lambda_{NL}}{\lambda_L} \) can be written as  
\[ \frac{\lambda_{NL}}{\lambda_L} = 1 + \frac{\lambda_T}{\lambda_L} \]  
(14)

The tensile load parameter \( \lambda_T \), because of large deflection is obtained from Eqs. (4), (10) and (15) as  
\[ \lambda_T = \frac{\pi^2}{4} (\frac{a}{r})^2 \]  
(18)

From Eq. (14), \( \frac{\lambda_{NL}}{\lambda_L} \) is obtained as  
\[ \frac{\lambda_{NL}}{\lambda_L} = 1 + b (\frac{a}{r})^2 \]  
(19)

**Results**

For a simply supported column the admissible function \( F(x) \) is taken as  
\[ F(x) = \sin \frac{\pi x}{L} \]  
(15)

which satisfies the boundary conditions  
\[ w(0) = w(L) = 0 \]  
(16)

By equating the strain energy and the work done expressions, we obtain the critical load parameter \( \lambda_L \) as  
\[ \lambda_L = \pi^2 \]  
(17)

The tensile load parameter \( \lambda_T \), because of large deflection is obtained from Eqs. (4), (10) and (15) as  
\[ \lambda_T = \frac{\pi^2}{4} (\frac{a}{r})^2 \]  
(18)

From Eq. (14), \( \frac{\lambda_{NL}}{\lambda_L} \) is obtained as  
\[ \frac{\lambda_{NL}}{\lambda_L} = 1 + b (\frac{a}{r})^2 \]  
(19)
Similarly for a clamped column, the admissible function taken is

\[ F(x) = \frac{1}{2} \left(1 - \cos \frac{2\pi x}{L} \right) \]  

(20)

which satisfies the boundary conditions

\[ w(0) = w'(0) = w(L) = w'(L) = 0 \]  

(21)

Following the same procedure for the case of a simply supported column, the value of \( b \), for a clamped column, is obtained as \( \frac{1}{16} \), with \( \lambda_L = 4\pi^2 \).

The values of \( b \) obtained from the present simple energy method are summarised in Table-1 along with those obtained by the finite element method and the Rayleigh-Ritz method for both the simply supported and clamped uniform columns. It may be noted here, that rigorous, nonlinear formulations are used in the case of the finite element and the Rayleigh-Ritz method. The present simple energy method gives the value of \( b \) exactly the same as the rigorous Rayleigh-Ritz method (also, the present results match excellently with those obtained by the finite element method, which is an analogue of the Rayleigh-Ritz method), thus showing the effectiveness of the simple energy method.

**Conclusions**

A simple energy method is proposed in this note to predict the thermal post buckling behaviour of uniform columns, with ends immovable axially. The numerical results obtained by using the present method are exactly the same as those obtained by the finite element method and the Rayleigh-Ritz method, using the rigorous nonlinear energy formulations for both the simply supported and clamped uniform columns. The method presented in this note is general, and can be applied to predict the thermal post buckling behaviour of columns with other boundary conditions and with complicating configurations, as long as the ends of the column are immovable axially.

**References**