FLIGHT TEST TECHNIQUES - AN OVERVIEW

S.C. Gupta
Department of Aeronautical Engineering
MVJ College of Engineering, Kadugodi Post, Near ITPL
Bangalore-560 067, India
Email : satish_chander_gupta@yahoo.co.in

Abstract

Flight test techniques towards performance, handling qualities, stability and control are brought out. Hazardous flight conditions are covered. Instrumentation aspects are highlighted. Data reduction and some flight evaluation cases are addressed. Airframe structural performance under different conditions of flight test are explained. Fault tolerant flight control system testing is brought out. Features related to online development of restructurable and reconfigurable control laws are highlighted.

Introduction

The objective of the flight testing is to collect data for verification of analysis performed on scaled models during wind tunnel testing and ground testing, and to verify actual aircraft capabilities in critical portions of the flight envelop. It is the ultimate tool to expand flight envelop by way of measurement of structural strains, estimation of stability and control derivatives, flying qualities assessment and propulsion parameters estimations. Flight dynamics testing emphasizes the evaluation of airframe structural performance under different conditions. Aircraft are subjected to various manoeuvres, flight profiles and load conditions to evaluate design integrity. The response of the airframe to different flight control stimuli is tested to verify that stress and vibrations do not result in unsafe conditions.

Manoeuvre longitudinal stability and load factor capability can be established through wind up turns and symmetric push-overs / pull-ups. The engine power and aircraft speeds need to be maintained constant. The demonstration of limits of flight and manoeuvre envelop requires exhibiting safe handling characteristics till envelop corner points. Flying qualities under degraded flight control system conditions, defining the relight envelops, working out the no spin recovery zone, and establishing e.g. travel limits which are allowable in complete flight envelop including resulting any emergencies needs to be recorded. Lateral and directional trim stability and controls are required to be demonstrated for safe handling. Rapid rolling and roll pullouts up to permitted stressing levels should be worked out. The maximum landings and most aft and forward c.g. points are required to be covered. The store loads should include: 1) highest mass, 2) highest pitch inertia, and 3) highest roll inertia. Any limitations in stores configurations are to be specified e.g. max permissible change in bank angle could be a limiting value of 180° or rapid rolling could be prohibitive in heavy stores. Rapid rolling with rudder coordination is generally sought to avoid sideslip. Safe store separation envelop needs to be defined through flight tests. Several types of uncommanded lateral directional motions are known to have been encountered at transonic speeds by high performance aircraft. There are three typical types of uncommanded motions at transonic speeds known as heavy wing, wing drop and wing rock. Heavy wing is mostly the result of a shock-induced trailing edge flow separation. This separation blankets the aft portion of wing thereby making aileron ineffective. If this happens asymmetrically between left and right wings, the aircraft will begin to slowly roll off as it is out of trim. Wing drop is much more abrupt in nature. It usually involves leading-edge flow separation or a rapid expansion of shock-induced flow separation occurring over the wing. Wing rock is closely related to wing drop, except that the motion is periodic and is limited in magnitude. It is largely marked by static lateral stability and the absence of roll damping.

The flow separation process, whether from shock/boundary layer interaction or leading edges or from hinge lines is viscous dominated and difficult to predict. The shock position over the wings can also be unsteady in the stall angle-of-attack region. Because of complexity of
flow, it is very difficult to resolve the problem during flight test. Studies conducted on free-to-roll wind tunnel models have shown very large solutions to transonic flow departure if the leading edge slat/flap are slightly drooped. The aileron-rudder input in a turn manoeuvre can result in wing drop followed by wing rock. Uncommanded lateral motions observed at transonic conditions can be of markedly different nature including aperiodic heavy wing caused by asymmetric trailing edge flow separation. Aperiodic abrupt wing drop caused by asymmetric wing stall, periodic limit cycle rock caused by loss of roll damping, and time dependent semiperiodic bursts of periodic motion caused by unsteady shock moments. Demonstration of transonic flight handling, dive recovery at such speeds, sustained flight at critical Mach number forms the part of flight handling.

Spinning of some fighter class of planes is required to be demonstrated. Spin tests are required to be demonstrated at maximum mass and practical loading with furthest forward and aft c.g. positions. Spin recovery action are explicitly written in the release to service documents. Number of permissible turns in a spin and safe heights for spinning and recovery action must be worked out. No spin recovery envelop needs to be established. Aircraft spin recovery tail parachute is a must for this purpose. During spin testing or high angles of attack manoeuvres or weapon firing cases, engine flame-out could occur. In-flight relight envelop in a single engine aircraft where engine starting has no provision for assisted start in flight is crucial to establish. In the event of engine flame out landing is to be executed otherwise the aircraft is lost. In the event of engine flame out all driven accessories make the control operations difficult or restricted. Engine out profile of descent from a low altitude to touch down with attempt to relight needs very careful planning. Choice of airfield and aircraft configuration must be planned for minimizing the risk. The hot relight is for the case where sufficient residual rotational speed is available, and the cold relight is for the case of compressor wind milling. The handling qualities of the aircraft in this case of power loss (Engine flame out condition) can degrade to level III and pilots workload may be very excessive. Tests points can be extremely flight critical. Sideslip in a roll manoeuvre further affects the air-intake performance.

Airplane configuration and state requires several parameters: flap setting, airplane position, undercarriage position, brake parachute and fuel state. Control inputs require stick position (pitch and roll), rudder displacement control forces, trim positions of all control surfaces. Signal sensor and engine parameters need to be monitored on line. Control laws need to be ascertained through flight testing. Current techniques aim to measure phase and gain margins of augmented aircraft during flight in real time and even reconfigure control laws on-line.

Performance Estimation

Level Flight

Level flight performance of Jet engines to assure design specifications and preparation of release to service documents involves in speed power test. The ratio of ambient to sea level pressure (δ) and that of temperature ratio (θ) in relation to the aircraft weight (W) and engine r.p.m (N) are of importance as shown in Fig.1. This figure shows the plots of constant W/δ lines for variations in N/\sqrt{\theta} and Mach number (M). Fig.2 shows the related thrust (T) versus refered RPM plots which are of importance [1, 2].

The gross thrust \( F_G \) can be determined by measuring the engine pressure ratio (EPR) and solving the equation (1) below.

\[
F_G/A_e = C_f (2\gamma/\gamma - 1) P_A \left( \frac{P_T}{P_A} \right)^{\gamma - 1} \left( 1 - \frac{1}{\gamma} \right) \tag{1}
\]

Where \( A_e \) is the exit area. \( \gamma \) is the ratio of specific heats, \( P_A \) is ambient static pressure and \( P_T \) is pressure at tail pipe exit at total temperature. The thrust coefficient (\( C_f \)) is known from the static ground calibration (Fig.3) typically shows a plot of \( C_f \) versus EPR i.e. \( P_T/P_A \).

Climb

Maximum range and maximum endurance are peculiar to air speed and thrust values. Maximum range speeds are always higher than maximum endurance because of the requirement of lesser drag at endurance. The best angle-of-climb airspeed is different than that of the best rate of climb airspeed. The earlier one is always a lesser value since the drag encountered is higher than the later one. Pressure altitude versus time history for various climb air speeds is obtained. Rate-of-climb versus Calibrated Air Speed (CAS) for different altitudes is thus known (See Fig.4). Climbs are repeated in opposite direction in order to cancel wind gradient acceleration effects on rate-of-climb. Also climbs are conducted in order to minimize wind effects. Tangent drawn parallel to CAS gives the best
rates of climb airspeed. Tangent drawn from origin provides best climb angle airspeed. The best rate-of-climb bears a constant proportion to best climb angle. However, compressibility effects could vary these proportions. Some more information is provided in Fig. 5 plot by way of non-dimensionalised climb rate. Climb performance method is another means of measuring thrust in flight. The change in true air speed with increase in altitude in climb performance is not catered in the formulas. The compressibility effects get accounted in the Indicated Air Speed. The energy approach to performance testing is based upon total energy (E) possessed by the aircraft, which comprises of potential energy (PE) and kinetic energy (KE) as below.

\[ E = PE + KE = mgh + \frac{1}{2} mV^2 \]  
(2)

where
- \( m \) = mass
- \( V \) = velocity
- \( h \) = height

By dividing through weight (W = mg)

\[ E_s = h + \frac{V^2}{2g} , \]

where \( E_s = E/W \), specific energy

\( E_s \) has the units that are of length and therefore, are referred to as energy height. Differentiating equation (2) w.r.t. time, result in \( \left( \frac{dE_s}{dt} \right) = \text{specific excess power} \) i.e. SEP, also given by equation (3) below:

\[ \frac{dE_s}{dt} = \frac{dh}{dt} + \frac{v}{g} \left( \frac{dv}{dt} \right) \]

\( = (F - D)\frac{v}{w} \)  
(4)

Lines of constant SEP \( (P_s) \) indicate region where thrust level regimes are high or low for manoeuvres. Plot of constant values of \( E_s \) and \( P_s \) are explained in Fig. 6. The path for minimum time-to-climb is traceable. In this figure if the lines of \( P_s \) are replaced by lines of constant \( P_L \) / fuel weight, then in a similar way, minimum fuel path can be traced. Fig. 7 shows constant drag lines plotted along with constant \( E_s \) lines. The maximum range glide can be spotted [1].

**Turning Performance**

Turn radius (R) and turn rate \( (\dot{\psi}) \) are important parameters for measurements. Turning performance gets limited by the i) thrust, ii) drag polar of aircraft, iii) \( C_{L_{max}} \) (maximum lift coefficient), and iv) structural strength. Typical \( C_{L_{max}} \) limitation boundary is shown in Fig. 8. Flight test method involves in recording on pilot’s data card the following information in a stabilized turn.

- Air speed and ambient temperature and pressure altitude
- Normal load factor \( (n_z) \) and bank angle
- Fuel consumption

The data is collected in the following way. At the test altitude, \( 'g' \) trim speed is maintained at full throttle. While maintaining altitude, \( 30^\circ \) bank is established and airspeed allowed to stabilize. The above data is recorded for various increasing bank angle values until the maximum \( n_z \) is achieved. The test is performed at several altitudes. Once the data has been obtained, it is reduced with the following procedure. The \( n_z \) is corrected to the aircraft test gross weight value. The plots of \( n_z/\dot{\psi}/R \) versus Mach numbers indicate peak performance in turn.

**Normalization**

In many cases, it may not be possible to obtain complete data for turning performance due to amount of data required. The following formula relates the \( n_z \) with net thrust \( (F_N) \).

\[ n_z \frac{W}{\delta} = F (M, F_N/\delta) \]  
(5)

From this relationship it is seen that for constant values of \( W, M \) and \( F_N/\delta \), the \( n_z \) depends upon \( \delta \). Thus turning performance at one altitude can be related to that at another altitude as below.

\[ n_z \left[ 1/n_z \right] = \delta_1/\delta_2 \]  
(6)

Altitude versus Mach number showing load factor and pressure ratio relationship are shown typically in Fig. 9 (indicating the equation 5 relationship).
Lift Drag Determination

A flight test technique employed to successfully verify the in-flight lift and drag is brought in Ref.2. Three classes of manoeuvres are used to establish the aerodynamic database; steady state, quasi - steady state and dynamic. Steady state manoeuvres (cruise, sustained turns) and quasi - steady state manoeuvres (accelerations, constant Mach climb) provide drag polar definition. Dynamic manoeuvres (roller coaster, constant Mach windup turns) allow evaluation of aerodynamic characteristics covering a range of angle of attack that cannot be achieved through steady-state or quasi - steady state manoeuvres [2].

Flight Path Angle Performance

The technique most commonly applied is the airspeed (V) versus flight path angle (γ) method. This method is based upon the approximation that for a given thrust, flight path angle is a function of airspeed. The V-γ relationship defines a level of excess thrust for each combination of V-γ. The steady climb technique aims to generate a plot of altitude versus time. The rate of climb (dH/dt) is related to flight path angle in the following way.

\[ \gamma = \sin^{-1} \left( \frac{dH}{dt} \right) / V_f \]  

(7)

Stability and Control Flight Testing

Static Longitudinal Stability

The stick fixed static stability (dc_m/dc_L) is related through pitch control deflection (δe) in the following way.

\[ \frac{d\delta e}{dC_L} = \frac{(dc_m/\delta e)_{\text{fixed}}}{Cm_{\delta e}} \]  

(8)

The \(d\delta e/dC_L \) is zero when \((dc_m/\delta e)_{\text{fixed}} \) is zero for a case of neutral point where aerodynamic center (a.c.) and center of gravity (c.g.) coincide. In case of stick free the dFs/dδe is zero around neutral point. However this is not a safe way to determine neutral point. A safer way to approach the problem is as following. Elevator position is measured for various equivalent air speeds for few c.g. positions ahead of neutral point. This applies for statically stable airframes. In case of unstable airframes or where flight control system (FCS) is augmented through fly-by-wire technology, the c.g. point could be anywhere. The lift coefficient at related values of c.g. locations is known from lift formula for against known weight of aircraft. The sequences in Fig.10 (a-c) indicate how neutral point can be determined. The expression for stick free longitudinal stability is governed by the elevator control force relationship. Fig.11 shows the stick force gradient (F_s) variations with airspeed after trim is set at various airspeeds. The test are [1,3] repeated for various static margin values.

Dynamic Longitudinal Stability

It is governed by the nature of phugoid and short period oscillations. During the phugoid, the altitude and airspeed vary, while the angle of attack remains constant, whereas during the short period oscillation, the altitude and speed remain nearly constant while the angle of attack varies. Flight test method for estimation of phugoid and data reduction is as follows. Aircraft is trim set at test speed and altitudes so decided. The speed is slightly disturbed by the use of pitch control. The phugoid airspeed versus time are determined. The amplitude ratio of this plot is known to relate to damping factor (ξ). The undamped natural frequency is governed by the expression below [1,3].

\[ w_p = \frac{2\pi}{\sqrt{1 - \frac{1}{\xi^2}}} \]  

(9)

Figure 12 typically shows the plot of envelop for a case of phugoid airspeed versus time.

Short Period Oscillation: Short period oscillations can be excited by a doublet input or through a pulse input or a gentle pull-up. The ‘g’ pull-up method results in a large amplitude input for short period motion with heavy damping. With the introduction of ‘fly-by-wire’ flight control system, new dynamic modes have been introduced, thereby changing significantly the airframe response due to pilots inputs. These responses are seen to be different than that of aircraft with statically stable airframes.

The close loop bandwidth frequency requirement is 6 dB gain and 45° phase margin for safe pitch handling. The open and close loop gain and phase values can be measured in flight. The pilot induced oscillations (PIO) are critically related by way of phase delay versus pitch attitude bandwidth. Bandwidth criteria boundaries in terms of PIO is |\( \phi \)|dB ≥ 1.5 rad / sec and phase delay of less than 0.15 secs for a case of no PIO. The second method usually applied is the Gibbons’s phase rate wherein the average phase rate AvPR defined by \( (d\phi/dt)_{\text{AvPR}} = 180^\circ \) is a measure of PIO. This value should be ≤ 100 deg / Htz for PIO.
avoidance for any phase angle crossover frequency. It is possible to predict configuration and points in flight envelope where PIO events are probable to occur. Three types of manoeuvres are commonly used for this purpose which are explained below [3].

**Attitude Tracking:** It involves in capturing and holding attitude angles (longitudinal and including that at various lateral bank positions). There is no FTI required in this case. A variant of this method has been created recently, consisting of two types of patterns, sum of sines and discrete inputs. The input signals shall have enough energy and bandwidth in order to excite PIO. Modeled turbulence can be stored and activated in the control loop to simulate turbulence. Power spectral density of signal form with high noise variations of small frequencies are alternative input concentrates and more powerful in low frequencies.

**Flight Path Angle Tracking:** This manoeuvre consists of flight path angle acquire and hold. The manoeuvre explores n ‘g’ lag q, i.e the way the longitudinal attitude begins to change immediately after longitudinal control input. The flight path angle will change after longitudinal control begins to change immediately after longitudinal attitude changes. The lag between n ‘g’ and q is proportional to $T_{gq}$ (secs) i.e. the flight path to attitude lag time constant. This manoeuvre requires the velocity display. PIO degrees must be established after each test point. At low velocities it is very high i.e the interchange between angle of attack and speed takes place at nearly constant load factor value, unless the power is added. Plots of $\Theta_{SP} \times T_{gq}$ versus $\zeta_{SP}$ are of great interest. Plots mark the region where PIO are susceptible.

**Off Landing:** Landing under cross wind conditions with larger sideslips and pitch excitation variations are complex and are severe combination for analysis hazardous during prototype development. Current criteria are exploring these phases of PIO. The flight test programme must choose the points to be evaluated in terms of PIO. Modern prediction method and in-flight display techniques of measuring gain and phase can be used to guide the choice process. A failure analysis must be conducted before actual flight test. PIO evaluation for every state of flight control system (normal, reversionary and back up mode) is necessary. It is time consuming to perform PIO evaluation in FCS failures with probability smaller than $10^9$ per hour of flight. PIO are common occurrence during development, certification and operation process of aircraft. New FCS architectures and features contribute to real time monitoring of phase lag between the pilots input and the aircraft response. Prediction methods such as Bandwidth and Gibsons’s Phase Rate are useful tools for PIO assessment.

**Lateral - Directional Static Stability**

Flight test methods include the following for obtaining manoeuvring stability data.

- Steady pull-ups and pushovers
- Steady turns at constant ‘g’ values
- Wind-up turns (‘g’ varying)

The stick force and n/\alpha terms describe the longitudinal stability criteria. The speeds need to be maintained constant since the measured stick force when trim airspeed is not maintained are vary different (Fig.13).

**Longitudinal Manoeuvre Stability**

This aspect of stability is assessed by measurements of the quantities of control displacements, forces, bank angle and sideslip in a steady heading sideslip (SHSS). To perform SHSS, aircraft is put into hands off trim at the test airspeed. SHSS is achieved though rudder deflection set at 1/4 full set increased to 1/2 and then 3/4 full in steps. At low speeds, large sideslip may result in one wing stall thereby resulting in snap roll departure. Aileron in opposite direction is used to hold wing level. Height loss may occur at low speeds therefore, this test should be started at safe height margins. Typical plots of rudder deflection, ($\delta r$), aileron deflection ($\delta a$), $\beta$ and $\phi$ in a SHSS are shown in Fig.14.

**Lateral - Directional Dynamic Stability**

Subject aspect is governed by the spiral and dutch roll mode. Fighter aircraft are designed to prevent spiral mode and have nominal dutch tendency. Dutch mode testing at high angles of attack is dangerous, since it may result in uncontrolled departure of the aircraft. Dutch roll mode is a lateral - directional oscillation, the rate of convergence, once excited, is of concern to pilot’s opinion on handling qualities of the aircraft. Dutch roll is a coupled motion, the roll to yaw, ($\phi / \beta$) is of importance to pilot. Damping parameter is another measure of dutch roll oscillation. It is linked to the number of cycles for the motion to damp to half amplitude ($C_{1,2}$). The damping parameter versus ($\phi / Ve$) governs the dutch roll specifica-
Hazardous Flight Testing

Stall, spin, flutter, heavy asymmetric store carriage test flying is larger risk involving. Stall / post-stall / spin flight test demonstration requirements for airplane are governed by MIL-S-83691A. Maximum allowable asymmetry should be demonstrated and FCS failure order during manoeuvre adequately addressed. Wing rock oscillations, bucking uncommanded pitching oscillations, nose slice i.e. uncommanded lateral / directional motion can occur during experimental flights. Roll attractor and wing rock around roll attractor are some more possible oscillations which can be present during high angles of attack testing.

The stall and normal acceleration during recovery are recorded. Uneven stall of wings leads to stick snatch. Use of boundary layer fence towards wing tips helps in uniformity of both wing stalls. If rolling and yawing are occur during recovery, these should be opposed by control application. The potential problem of the wing drop / roll-off is due to the differences in contour (shape) of the right and left wing. Wing leading edge shapes are extremely detrimental to wing drop, especially where leading edge skin is formed by dies. Deep stall lock-in may occur where excessive down loads occur on elevators due to shed vortices resulting in reduction in elevator pitch down capability to reduce angle of attack for stall recovery. The stalls are demonstrated at specified deceleration rate prior to stall.

Spin when fully developed is either upright / erect or inverted. In the later case the yawing and rolling motions are in opposite direction. Rotary motion of spin may have oscillations in pitch, roll and yaw superimposed upon them. Incipient spin is the initial, transitory phase of motion during which it is not possible to identify spin mode. Fully developed spin is attained when the trajectory has become vertical and no significant change is noted in the spin characteristics from turn to turn. Post-stall gyration i.e uncontrolled motion about one or more aircraft axes following departure at high angles of attack can be also be present. The c.g. envelop for spin testing and the control action for spin are explicitly worked out. In the aerobatic category, the recovery from spin in a fully developed spin should be possible in one and one half additional turns by the normal use of controls, without exceeding airspeed or load factor. In case of flap down spin test, if required it should be permissible to retract the flap during recovery. Spin characteristics requirements are specified for fighter aircraft design. The recovery is worked out and demonstrated. Spin recovery parachutes are installed on aircraft during prototype testing. The spin test matrix is adequately specified for flight test programme. An ideal spin recovery should be possible by off loading the angle of attack. The use of rudder generates unfavorable rolling moment which is prospin. The need for out spin aileron action along with pitch control for spin recovery vis-à-vis the c.g. envelop and cases of store asymmetries should be established for varied altitude conditions. Autorotation can occur if one of the wings is under stalled conditions while the other is not. It is a vertical gyration motion, if not controlled in time, can dangerously develop into uncontrollable attitude.

Wing rock can occur on swept back wings at high angles of attack. Wing rock is a limit cycle roll oscillation experienced by aircraft where amplitude and frequency of wing rock is a nonlinear function of many parameters such as angle of attack, sideslip etc. Several theories have been put forward over the years to explain the wing rock phenomenon. Some of the factors, which emerge out of these are as follows : 1) wing rock in initiated because of vortex asymmetry, 2) vortex bursting does not initiate wing rock, but plays an active part in limiting the amplitude of the limit cycle, 3) there is negative roll damping at small angles of bank and positive roll damping at higher angles of bank. 4) wing rock is caused by the relative time lag between the static and dynamic position of vortex normal to the wing surface. These studies indicate that the vortex formation plays important role during wing rock, hence manipulation of these vortices help in wing rock suppression. Wing rock around roll attractor is structurally overloading and unsafe condition since it is not normally accounted for in the detailed structural design. Flight test condition should be able to foresee from parametric estimation the likely presence of such phenomenon and account in design work. Certain manoeuvre conditions are load critical which otherwise might not be foreseen during design and development. The rudder doublet action during a velocity axis roll generates loads which are more severe than rapid roll case or case of steady heading sideslip. The strain gauging of critical load areas need careful workout.
Flutter

Flutter flight test are done to determine V-n envelop free of flutter. The coupling of resonant frequencies of various modes create flutter. The most common approach to flight flutter testing is to track estimated modal damping ratios of the aircraft over a number of flight conditions. These damping trends are then extrapolated to predict whether it is safe to move to the next test point and also to determine the flutter speed. During the design stage of the aircraft the aeroelastic behavior is estimated by producing detailed mathematical and aeroelastic wind tunnel models. Subcritical damping data can not always be safely extrapolated to obtain accurate prediction for flutter velocity. The aeroelastic stability can change from positive to negative with little increase in value of airspeed. There are several flutter prediction methods. The following five methods are described herein [4].

- Damping ratio variation with airspeed
- Flutter margin method
- Envelop function
- Nissim and Gilyard method
- Autoregressive moving average - based method

Damping Ratio Variations with Airspeed: It indicates the flutter instability. At flutter, the damping is zero, thereby causing self excited oscillations. There are several system identification methods in time as well in frequency domain. The damping ratio trends are estimated and curve fitted to extrapolate the estimated flutter velocity.

Flutter Margin Method: It employs the Routh stability criterion. The response to a known input at a subcritical air speed is recorded and eigenvalues of the system are calculated. The flutter margin is seen to be a quadratic function of the dynamic pressure \( q \) as below.

\[
F = B_2 q^2 + B_1 q + B_0
\]

where \( B_0, B_1 \) and \( B_2 \) are coefficients to be evaluated. The flutter condition is \( F=0 \). To cater for the experimental uncertainty, the flutter margin is estimated at a wider range of subcritical air speeds and then fitted in a least square way.

Envelop Function Method: The basis of method is that impulse response of any stable damped system is decaying, with the shape of the decay in time domain being described by the decay envelop. As an example, for the impulse response \( y(t) \), the decay envelop is given by equation (11) below.

\[
env(t) = \sqrt{y^2(t) + y_H^2(t)}
\]

where

\[
y_H(t) = F^{-1} [\text{Im} \{ y(w) \}] - j \text{Re} \{ y(w) \}
\]

\( y(w) \) is the Fourier transform of \( y(t) \) and \( \text{Im} \) and \( \text{Re} \) are imaging and real time parts. The time centroid of the decay envelop is given as below:

\[
\bar{t} = \frac{\int_0^{t_{\text{max}}} env(t) \, dt}{\int_0^{t_{\text{max}}} env(t) \, dt}
\]

The \( \bar{t} \) increases as damping reduces. \( \bar{t}^{-1} \) employs as the significant shape parameter i.e., \( S = \bar{t}^{-1} \). The value of \( S \) for flutter condition is \( S = 2/t_{\text{max}} \). The procedure involves in estimation of \( S \) at number of subcritical speeds. The \( S \) versus airspeed curve is extrapolated to the point where \( S = 2/t_{\text{max}} \) which results in flutter velocity. The impulse response of an aeroelastic system can be obtained by giving a stick jerk as impulsive excitation.

Nissim and Gilyard Method: It includes the aerodynamic loading versus Mach number in the following way. The equations of motion for a forced aeroelastic system are:

\[
\bar{M} \ddot{\bar{q}} + \bar{C} \dot{\bar{q}} + \bar{K} \bar{q} = \bar{F} g(t)
\]

This equation after multiplying with, \( M^{-1} \) is transposed into frequency domain as equation (14), which after rearrangements becomes equation (15).

\[
[-I \, w^2 + c_j \, w + k] \, q(w) = F_g(w)
\]

where

\[
c = \bar{M}^{-1} \bar{c}, K = \bar{M}^{-1} \bar{k}, F = \bar{M}^{-1} \bar{F}
\]

\[
C_j \, w \, H_q(w) + K \, H_q(w) - F = I \, w^2 \, H_q(w)
\]
where \( H_q(w) = q(w)/g(w) \)

Knowing the structural part and aerodynamic part, the equation (15) is solved in frequency response domain for known aerodynamic loading. Flutter speed can be determined by iterative calculations. More accurate aerodynamic loading is known from experimental data.

**Auto Regressive Moving Average (ARMA) - Based Method:** In this, equations of motion of a dynamic system can be expressed as a sum of the regressive response terms equal to the value of the regressive forcing term. The AR part is the term containing the response \( y \) and MA part denotes the excitation disturbance noise term \( \mu \). In general form the ARMA equation can be written as below, where \( J \) is the number of modes of the system including noise effects. The \( a, b, \) and \( J \) are evaluated using a parametric based algorithm. The resulting matrix equation can be solved and the eigenvalue of the system can be obtained by forming the following characteristics polynomial, with roots governed by equation (17).

\[
G(\mu) = \mu^J + b(2J-1)\mu^{J-1} + \ldots + b(1)\mu + b(0) = 0 \quad (16)
\]

\[
\mu_{1,2} = \exp\left((-\gamma_0 \pm j\alpha \sqrt{1 - \zeta^2})\Delta t\right) \quad (17)
\]

A case of typical flutter test points are shown in a flight envelop for progressive clearance of envelop of a fighter aircraft configuration (Fig.16). Some results of frequency versus speed for various modes and damping are shown graphically in Fig.17.

There are recent on-line tools known as ‘Flutterometer’ that indicate a measure of distance to flutter in terms of flight condition during the flight test. The measure of distance is computed as a flutter margin by applying analysis which is considered the worst case effects of modeling uncertainty. Flutterometer is a model-based tool. The flutter margin is predicted by analyzing the stability properties of an analytical model. Flutterometer is a state-space model-based analysis tool. The structure and unsteady aerodynamics are modeled and then a state-space model using rational functional approximation is made. Structural dynamics can be very sensitive to weight conditions. The data on fuel weight, the flight parameters are inputs to flutterometer [5]. The structure model should be augmented to include the excitation and sensing elements.

**Flight Testing Large Lateral Asymmetries**

The risk reduction using real time analysis is suggested while flight testing large lateral asymmetries. The problem of asymmetric loading can be illustrated by examination of the generalized equations (18) of motion for roll, pitch and yaw moments for a rigid body as below which are for a c.g. case of lateral symmetry.

\[
\begin{align*}
I_{xx} & = I_{xx} \dot{p} - I_{xy} \dot{q} - I_{xz} \dot{r} - I_{xz} \dot{p} + I_{yy} \dot{q} + I_{yz} \dot{r} \\
I_{xy} & = I_{xy} \dot{p} + I_{yy} \dot{q} + I_{yz} \dot{r} - I_{xz} \dot{p} + I_{yy} \dot{q} + I_{yz} \dot{r} \\
I_{xz} & = I_{xz} \dot{p} + I_{yz} \dot{r} - I_{xz} \dot{p} + I_{yy} \dot{q} + I_{yz} \dot{r} - I_{xz} \dot{p} + I_{yy} \dot{q} + I_{yz} \dot{r} \\
\end{align*}
\]

where \( l, m, n \) are roll, pitch and yaw moments.

There are at least two of flight mechanics issues associated with asymmetric loading that should be considered. 1) cross axis coupling resulting from lateral shift in c.g. position and the resulting non-zero products of inertia \( I_{xy} \) and \( I_{yz} \), and 2) the possible aerodynamic effects of the store asymmetry. Additionally, the presence of an asymmetry causes pilot inputs in one axis to perturbate all three axis, thus complicating the performance of precision handling qualities. The release of heavy stores is done simultaneously. In case of hang-up there would be sudden change in handling qualities which need to be addressed before clearing the fleet to deliver these large weapons. The cross-axis coupling resulting from asymmetric loading is to be controlled by the aerodynamic restoring moments. Determining the maximum control surface deflection schedules can provide significant insight into the available control power. To understand the implications of the control schedule commands, information on the maximum control authority available across the flight envelop need be known.

The aileron command normally used for rapid rolling in case of light store carriers are scheduled with angle of attack and air data to improve coordination at high angles of attack and low dynamic pressures, and also minimize the hinge moment limiting, wing flexibility, and roll reversal at high dynamic pressure. In a typical case of differential tail plane (Diff-TP), the diff-TP action is limited by the pitch command and load factor, as well as, alleviates the bending moments. Differential leading edge action is normally avoided at design stage in favor of lesser complicated flight mechanics. Maximum lateral control needs to be generated. The rolling moment generated by the store carried at a distance \( \gamma_{STO} \) from x-axis is :
Fault Tolerant FCS Testing

The departure risk is hazardous to flight with large lateral weight asymmetries which needs to be ascertained. The asymmetric drag requires increased rudder displacements to maintain maximum sideslip which imparts higher normal loads on vertical fins. The second concern involves structural loads applied to the outboard pylon resulting from higher roll rates that may be experienced when rolling towards the heavy wing. The absence of store on a pylon in asymmetry shall result in high rolling rates since FCS will sense absence of store and this will result in larger structural loads than planned on pylon with mounted store on one side. If the pilot then applies a full opposite stick command to stop the roll, the structural loads on pylons can be significantly higher than normal, especially at elevated load factors.

\[
L_{STO} = n_z \times W_{STO} \times y_{STO}
\]

(19)

The mathematical model of an aircraft has sustained damage to a primary control surface. Starting a PID process following a failure damage is a potentially a very critical issue. The mathematical model at post-failure conditions is used by a failure accommodation scheme to compute on-line the compensating control signals to command the remaining healthy control surfaces for a safe continuation and termination of the flight. A fault tolerant FCS is required to perform failure detection, identification and accommodation for sensor and actuator failure. Failure accommodation strategy should include a variety of control surfaces (speed brakes, wing flaps, control spoilers, differential tail plane etc), as well as thrust mechanisms. The need of introducing fault tolerant capabilities for a relaxed static stability aircraft with a flight envelop up to nonlinear angle of attack requires an increase in the number of independent control surfaces. The selection of the control mechanism to be used is a function of several factors: control effectiveness, increased aircraft complexity, weight penalties, drag criteria etc. For a fault tolerant control logic to be effective, the traditional flight data, the actuator position for each surface, along with a fully operational flight computer are to be available. Failure and / or battle damage of a control surface can be classified in two categories- locked surface and missing surface. A locked surface is associated with a mechanical failure in the actuator; a battle damage, mainly implies a missing surface, rather both, a missing and locked surface. Dynamic coupling between longitudinal and lateral-directional dynamics following any type of control surface failure in addition to non-linear dynamic and aerodynamic condition is of concern. This may lead to loss of stability. Following an actuator failures the objectives of a fault tolerant FCS are to achieve: 1) a lower damage-induced handling qualities, 2) a lower mission abort rate, 3) a lower aircraft loss rate [6].

Within fault tolerant control schemes, one can distinguish between reconfigurable and restructurable approaches. A necessary condition for both classes of fault tolerant approaches is that the aircraft system is still controllable and therefore, trimmable following system failure / damage. Within reconfigurable flight control systems a complete set of possible failures for different control surfaces is defined apriori the relative changes in control gains are then calculated off-line - using one of several control approaches - and stored in an on-board memory, ready for on-line use whenever needed. A disadvantage of this approach is the required apriori extensive design to accommodate all possible control system failures at different conditions in the aircraft flight envelop. This in turn, requires extensive memory space in the on-board computer. Restructurable fault tolerant systems are conceptually different. Within these the reconstruction of the control laws is instead performed on-line. In this case the availability of computational power, as opposed to memory, could be more critical. Restructurable control laws are formulated using on-line estimates of aircraft parameters from a real time PID scheme. On line PID, particularly within an adaptive control schemes, is a more challenging task. A basic assumption for on-line PID is a modeling of the dynamic system with time-varying parameters, which can be aerodynamic parameters, conditions, inertial properties etc. Current research activities are towards restructurable fault tolerant scheme based on failure accommodation phase. For example formulation online to regain control of the aircraft at the non linear dynamic conditions is one phase. Second phase is the once to back stable and linear conditions, a linear restructurable control law is formulated.

Accurate post-failure aerodynamic models are needed for the simulation purposes. The aerodynamic characteristics of a surface are to be expressed in terms of normal force, axial force and moment around control point. The longitudinal failure is more important than the aileron and rudder failures because of the unavoidable coupling between the longitudinal and lateral - directional dynamics. On-line time-domain PID techniques mainly include vari-
ations of the least square regression method, such as recursive least square. Potential problem with the time domain PID techniques may be the lack of a reliable parameter for an on-line estimates in the presence of unmodeled noise. To overcome these problem, the frequency - based PID techniques using Fourier transforms are useful. Aircraft dynamics is modeled using the conventional continuous - time state variable model [7]. In a multiple redundant FCS, characterization of failure transients is required to contain failure transients under failure conditions. Detailed modeling and analysis to allow in-depth investigations to predict aircraft transients due to failures and to evolve subsequent system reversions is not only tough to do with perfection but the whole process is generally not adequate to provide enough confidence for clearance. Hence the design is based on quasi static calculations of failure transients and compliance with the requirements is validated by testing on the real-time simulators. Failures are injected into real systems by inputs of several types like step, pulse, ramp and cyclic variations to stimulate the system in open loop. Failure transients of control surfaces under simulated failure conditions are thus characterized and final evaluation is done through pilot in closed loop. Hardware in loop simulation facility is the only ultimate platform in clearance process.

Fighters aim at carefree manoeuvring for relieving pilots work load. Envelop protection for carefree manoeuvring is governed by limit detection and avoidance, and aircraft constrained by structural, power and control margins etc. Limit avoidance if left to pilot results in pilot’s increased work load and simplified operational limits results in sub optimal performance. Possible benefits with a limit avoidance system are following. The key to limit avoidance is a timely prediction of the limit margins and its translation to control / command margins.

- Improved agility and manoeuvrability by expanding the usable flight envelop
- Improved handling qualities by providing carefree handling
- Reduced maintenance costs by avoiding excessive limit violation
- Improved flight safety by warning the pilot when limits are approached

**Instrumentation Aspects**

**Aerodynamic Probe**

An aerodynamic probe installed on the aircraft measures a local angle of attack and sideslip angles ($\alpha_{\text{loc}}$, $\beta_{\text{loc}}$). These are to be related to the aircraft angles at the c.g. location, ($\alpha_a$, $\beta_a$). The aerodynamic velocity of the probe is governed by the expression below:

$$V_{a,s}^b = \begin{bmatrix} u_{a,s}^b \\ v_{a,s}^b \\ w_{a,s}^b \end{bmatrix}$$

(20)

Flow angle can be calibrated using inertial navigation system of the aircraft. The probe measurements should be converted to the c.g. location of aircraft.

An example of test specific in nature for flight test is as brought out below for flow angle calibration using inertial navigation system.

The flow angle calibration can be done by calibration from dynamic manoeuvres. The analysis technique involves computation of flow angles using the ground speed ($V_N$ - North velocity, $V_E$ - East velocity, $V_D$ - down velocity), Wind velocities ($V_{WN}$, $V_{WE}$) and the Euler angles ($\phi$, $\theta$, $\psi$) as explained below.

From North, East and down velocities, one can derive the true air speed components in the earth fixed references frame, by applying the wind corrections as follows:

$$V_r = V_N - V_{WN}$$

$$V_l = V_E - V_{WE}$$

$$V_d = V_D$$

(21)

Here the vertical component of wind velocity is assumed to be zero. As such the trial is to be conducted in calm weather. Further, the analysis assumes that the wind velocity remains constant during the manoeuvre. There are two methods in vogue for estimating the winds. In one technique the aircraft is held in steady level flight prior to the manoeuvre in order to estimate the wind velocity components. The second technique is slightly different, in that the wind velocity components are estimated during the manoeuvre by the following process [8].
\[ V^2 - (V_N - V_{WN})^2 + (V_E - V_{WE})^2 + V_D^2 \]  \tag{22}

The sum of squares of residual errors in TAS over the whole manoeuvre is therefore as below:

\[
\text{ERRSUM} = \sum_{j=1}^{n} [ (V_j^2 - (V_{Nj} - V_{WN})^2 - (V_{Ej} - V_{WE})^2 - V_{Dj}^2 ) \]  \tag{23}

Values of the (unknown) wind velocity components \( V_{MN} \) and \( V_{WE} \) need to be chosen such that the error sum is a minimum. At this minimum, partial derivatives of the error sum w.r.t. each of the wind velocity components will be zero. That is as below:

\[
\frac{\delta \text{ERRSUM}}{\delta V_{WN}} = \sum_{j=1}^{n} [ V_j^2 - (V_{Nj} - V_{WN})^2 - (V_{Ej} - V_{WE})^2 - V_{Dj}^2 ] \times (V_{Nj} - V_{WN}) = 0 \]  \tag{24}

and

\[
\frac{\delta \text{ERRSUM}}{\delta V_{WE}} = \sum_{j=1}^{n} [ V_j^2 - (V_{Nj} - V_{WN})^2 - (V_{Ej} - V_{WE})^2 - V_{Dj}^2 ] \times (V_{Ej} - V_{WE}) = 0 \]  \tag{25}

Equations (24) and (25) may be solved by a two dimensional Newton-Raphson iteration procedure.

The earth fixed TAS components are then transformed to body axis by using the following transformation matrix.

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} = 
\begin{bmatrix}
  \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
  -\cos \phi \sin \psi + \sin \theta \sin \phi & \sin \phi \sin \psi + \cos \theta \sin \phi & \cos \phi \cos \psi \\
  \sin \phi \cos \psi + \sin \theta \cos \phi & \sin \phi \sin \psi + \cos \theta \cos \phi & \cos \phi \sin \psi - \sin \theta \cos \phi
\end{bmatrix}
\begin{bmatrix}
  V' \\
  V' \\
  V'
\end{bmatrix}
\]

\tag{26}

Where \( \theta, \phi, \psi \) are pitch angle, bank angle and yaw angle in Eulers conditions. The following correction for aircraft rates of rotation are required.

\[
\Delta u = qz - ry \\
\Delta v = rx - pz \\
\Delta w = py - qx
\]  \tag{27}

where \( x, y, z \) are the longitudinal, lateral and vertical distances from the aircraft center of gravity to the inertial platform. Separate rate gyro package will be needed to record the aircraft rates of rotation with sufficient accuracy.

Finally,

\[
V^2 = u^2 + v^2 + w^2
\]

\[
\alpha = \sin^{-1} \left( \frac{w}{\sqrt{u^2 + w^2}} \right) = \tan^{-1} \left( \frac{w}{V} \right)
\]

\[
\beta = \sin^{-1} \left( \frac{v}{V} \right)
\]

Load Factor Sonde

The measurement of acceleration is equivalent to a weighing of a mass \( m_{SA} \). The accelerometer is an unsteady state with a given mass to weigh. This given mass \( m_{SA} \) can be denoted the subjected mass because this mass is subjected to the reaction of the accelerometer \( R \) and to the gravitational attraction \( g_{r,SA} \) at the accelerometer sonde (SA) location. The accelerometer device could be called the sonde. The fundamental equation of mechanics leads to following [9].

\[
m_{SA} A_{I,SA} = m_{SA} g_{r,SA} + R
\]  \tag{31}

Physically, the accelerometer experiences the reaction \( R \) and this parameter is provided by the measurement. As the subjected mass is known, the measurement obtained offers the value of:

\[
\text{Accelerometer measurement} = A_{I,SA} - g_{r,SA} \]

\tag{32}

with the vector \( A_{I,SA} \) equal to the inertial acceleration at the sonde station SA. From this result, the concept of load factor \( n \) can be introduced. The load factor is equal to the ratio of the accelerations to the value of gravitation \( g_{r,SA} \). The gravitational attraction \( g_{r,SA} \) can be considered as an acceleration.

\[
n_{SA} = \frac{g_{r,SA} - A_{I,SA}}{g_{r,SA}}
\]  \tag{33}

Prescribing the load factor \( n \) is a vector so that it has three components depending on the projection frame. Usually the term load factor is also associated with the \( z \) component on \( n \) in the aerodynamic frame \( F_a \).
form for the definition of \( n \) is the ratio of the massic forces to the weight value. The massic forces are the weight and the inertial force that is to say all the terms of the fundamental equation of Mechanics which explicitly depends on the mass. The inertial force \( -mA_G \) is the opposite of the mass multiplied by the inertial acceleration, so that this inertial force (I) can be inserted in the force class. Thus,

\[
\begin{align*}
n \_ G &= \frac{m g_{r, G} - mA_G}{m, g_{r, G}} \\
\end{align*}
\]

(34)

the fundamental equation of Mechanics can be written as

\[
0 = N \_ G m g_{r, G} + F _{aero} + F _{thrust}
\]

(35)

With this last relation, a new expression of the load factor is found depending on the aerodynamic force \( F _{aero} \) and the propulsive force \( F _{thrust} \).

\[
\begin{align*}
n \_ G &= \frac{F _{aero} + F _{thrust}}{mg_{r, G}} \\
\end{align*}
\]

(36)

Global Positioning System (GPS)

GPS can be utilized for navigation equipment testing. The ground speed and ground track can have significant errors. Variations in wind directions and velocity results in position error. GPS position error therefore needs to be established. Three perpendicular GPS track at the same indicated air speed can provide information on true air speed. This horse shoe heading method is explained in Fig.18, where \( V_T \) the true airspeed, \( v_w \) is westerly component of wind and \( v_N \) is northerly component [9].

Three equations for three unknowns \((V_T, V_N, V_W)\) can be written as below, where \( V_1, V_2, \) and \( V_3 \) are measured known ground GPS speeds for three headings.

\[
\begin{align*}
(V_T - V_N)^2 + V_W^2 &= V_1^2 \\
(V_T - V_W)^2 + V_N^2 &= V_2^2 \\
(V_T - V_N)^2 + V_W^2 &= V_3^2
\end{align*}
\]

(37)\hspace{1cm}(38)\hspace{1cm}(39)

Totaling equations (37) and (39), equation (40) results:

\[
2V_T^2 + 2V_N^2 + 2V_W^2 = V_1^2 + V_3^2 = AX
\]

(40)

Subtracting equation (39) from (equation (38), the following results:

\[
-2V_T V_N + 2V_T V_W = V_3^2 - V_2^2 = BY
\]

(41)

Subtracting equation (39) from equation (37), the following results:

\[
-4V_T V_N = V_1^2 - V_3^2 = CZ
\]

(42)

or \( V_T V_N = -CT/4 \)

(43)

Since \( V_1, V_2, \) and \( V_3 \) are known. Therefore, \( AX, BY \) and \( CZ \) are known. Equations (41), (42) and (43) with three unknowns \( V_w, V_N, \) and \( V_T \) can be solved. Using equation (43), equation (41) results in following:

\[
2V_T V_w = BY - CZ/2
\]

(44)

or \( V_w = (BY - CZ/2) / 2V_T \)

(45)

From equation (42)

\[
V_N = -CZ/4V_T
\]

(46)

Using equation (45) and equation (46) in equation (40), true air speed can be obtained from equation below:

\[
V_T^2 - (AX/2) V_T^2 + 2 (BY^2 - 2CZ \times BY + CZ^2) = 0
\]

(47)

Thus

\[
V_T = \frac{-b \pm b^2 - 4ac}{2}
\]

where

\[
b = AX/2, c = (BY^2 - 2BY \times CZ + CZ^2)
\]

(48)

In-flight Flow Visualization

Flow cones, tufts, oil flows, liquid crystals, sublimating chemicals, and emitted fluids are useful as flow visualization methods. Off-surface flow visualization of vortical flow can be achieved through natural condensation or smoke generator system. Boundary layer transition techniques utilize oil flows and liquid crystals, infrared imaging and sublimating chemicals oil flows and liquid crystals are used to visualize shock waves. Flow direction
and boundary layer separation visualization techniques utilize flow cones and tufts [9].

Infrared imaging requires extensive hardware, including liquid nitrogen cooled imager and display. Infrared imaging is the only non-intensive method. Liquid crystal are currently the most common method of detecting the laminar to turbulent transition location through flow visualization. These provide favourable results and are capable of continuously and quickly reacting to flow changes, allowing multiple test points to be obtained per flight. Dust tends to adhere to glove surfaces coated with liquid and can contaminate results, uneven thickness in liquid crystal coat can cause changes of colour of the liquid crystal pattern, interfering with interpretation of the patterns. The location of transition also gets affected by the Reynolds number effects.

References
Fig. 3 The $C_1$ Variation Pattern

Fig. 4 Maximum Best Climb Angle Airspeed

Fig. 5 Typical Non-dimensional Climb Rate

Fig. 6 Minimum Time-to-Climb Path

Fig. 7 Maximum Range Glide Path

Fig. 8 Maximum Manoeuvre Criteria
Fig. 9 Load Factor Versus Pressure Relationship

Fig. 10a $\delta_e$ Versus $V_e$ for Various c.g. Position

Fig. 10b $\delta_e$ Versus $C_L$

Fig. 10c Locating Natural Points

Fig. 11 Altitude Versus Airspeed Plots

Fig. 12 Phugoid Airspeed Versus Time

Fig. 13 Effect of Trim Airspeed being not Maintained While Collecting Stick Force Data
Fig. 14 The Aileron and Rudder Deflection are in Opposite Direction in SHSS

Fig. 15 Rapid Roll of a Medium Laterly Stable Configuration

Fig. 16 A Case of Flutter Test Points

Fig. 17 Typical Flutter Frequencies

Fig. 18 Horeshoe Heading Method